

# Non-local spin valve in Van der Pauw cross geometry with four ferromagnetic electrodes.

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## Abstract

We consider a non-local spin valve in a Van der Pauw cross geometry with four ferromagnetic electrodes. Two antiparallel ferromagnets are used as (charge) source and drain while the detector circuit involves measuring the voltage between two collinear ferromagnets with parallel or antiparallel magnetizations. We find a potentially large increase of the non-local spin voltage. The setup displays several additional interesting properties: (i) infinite GMR for the non-local resistance (if a symmetry requirement for the device is met); (ii) ON-OFF switch effect, when the injector electrodes are parallel instead of antiparallel; (iii) insensitivity to offset voltages. The device can additionally be used as a Direct Spin Hall Effect probe and as a reprogrammable magneto-logic gate implementing basic operations (NOR, NAND, inverter, AND, OR, etc).

## I. INTRODUCTION.

Pure spin manipulation is an important topic in spintronics due to possible applications for programmable logic and memory. Non-local spin valves<sup>1-6</sup> provide an example of pure spin current generation. In the latter a pure spin current generated at a ferromagnetic - paramagnetic interface reaches a ferromagnetic probe in the absence of charge current; the spin accumulation created in the probe vicinity can then be detected as a charge voltage by virtue of Johnson-Silsbee charge-spin coupling<sup>1,7,8</sup>. Such pure spin currents have already proved useful to switch magnetizations<sup>9,10</sup>. This is opening promises for further applications in logic, sensing and memory devices but to that end it is desirable to increase the signals: the spin voltages are typically in the  $\mu V$  range while  $mV$  would be more suitable to ensure sufficient  $SNR$  (signal to noise ratio).

While spin valves are in everyday use in hard-drives, in order to reach or even go beyond the  $1\text{ Tbit}/inch^2$  density in hard-drives<sup>11-13</sup>, novel spin valves are required with a low  $RA$  resistance times area product (typically  $RA \leq 0.1\ \Omega\mu m^2$ )<sup>12-14</sup> while sustaining a  $mA$  current and  $mV$  voltage; this is beyond TMR (tunneling magnetoresistance) sensors capability since they are too resistive. Metallic spin valves are therefore more appropriate. However in order for them to have a suitable  $SNR$  it is also necessary that the read-heads function with a large enough contrast<sup>12</sup>:  $\Delta RA \geq 5\ m\Omega\mu m^2$ . Metallic spin valves with larger GMR ratio  $\Delta R/R$  are therefore required.

At first sight non-local metallic spin valves are not obvious candidates for larger MR (magnetoresistance) ratios: they indeed underperform when compared to their local counterparts; using the same materials and dimensions a local  $CPP$  spin valve is expected to have a larger  $\Delta RA$  since spin confinement is better<sup>15-17</sup>.

The goal of this paper is to discuss a non-local spintronics device with potentially:

- enhanced spin voltage in the  $mV$  range for currents  $\sim mA$  (so that the non-local resistance variation  $\Delta R_{nl}$  is in the Ohm range) with realistic density currents  $j < 10^8\ A/cm^2$  addressing the needs of industry.
- enhanced non-local  $GMR$  ratio:  $\Delta R_{nl}/R_{nl}$  (up to 100% for the pessimistic ratio; or up to infinity for the optimistic ratio), helping quite generally for better SNR and perhaps making them suitable candidates as sensors or read-heads for hard-drive areal

densities larger than  $1 \text{ Tbit/inch}^2$ .

Regarding the enhancement of the non-local signal, spin valves with tunnel junctions have been reported with non-local resistance in the  $\Omega$  range but due to a polarization decreasing rapidly when the current is larger than  $\sim \mu\text{A}$ , the spin voltage remains small in the usual  $\mu\text{V}$  range<sup>18,19</sup>. However much progress has been reported recently in pure metallic lateral spin valves ( $\sim 10 \mu\text{V}$  for nanopillars<sup>10</sup>) or lateral valves with very low resistance tunnel junctions (using a thin  $\text{nm}$   $\text{MgO}$  layer)<sup>20–22</sup>, reaching in the latter case the  $100 \mu\text{V}$  range with  $RA \sim 0.2 \Omega\mu\text{m}^2$  so that already  $\Delta R A$  is of the order of a few  $1 \text{ m}\Omega\mu\text{m}^2$ .

We propose to go even further in the improvement by relying on two ideas: (i) use two injectors instead of a single one, which should at face value double the signal; (ii) enhance the spin confinement by making good use of tunnel barriers, thin enough to stay close to the metallic regime but resistive enough to hinder spin leakage. The idea of minimizing the spin relaxation volume has been expressed in particular in<sup>17,23</sup> and explains the large signals seen in spin valves using carbon nanotubes<sup>24</sup>.

Our basic setup applies these ideas by using four collinear ferromagnetic terminals. In a standard lateral spin valve<sup>2,3,25</sup> the charge current flows from a ferromagnetic electrode to a paramagnetic drain; the injector electrode is connected by a lateral wire to another ferromagnetic electrode used as a detector. In our setup we propose to replace the paramagnetic drain by a ferromagnet antiparallel to the terminal acting as current source; the two antiparallel electrodes are connected by a paramagnetic metal with thin tunnel barriers in order to better confine spin. The two antiparallel ferromagnets act as spin sources although in terms of charge one is a source and the other a drain: this effectively doubles the spin accumulation in the lateral wire while the tunnel junctions make sure spin is confined.

We further change the standard detection setup by using a ferromagnetic counter-electrode instead of a paramagnetic one. The advantage of using two ferromagnets is evidenced when the two detector electrodes are placed symmetrically with respect to the injectors (source and drain): provided they are otherwise identical terminals this implies that when their magnetizations are parallel, their voltage difference should be identically zero by symmetry. This is how we reach an infinite non-local  $GMR$  ratio.

We will also address the issue of voltage offsets plaguing non-local setups<sup>1,2,4,5,15,16</sup>: while voltages generated by spin accumulation are clearly observed, some additional voltages of various origins are also usually seen. These offset (or baseline) voltages have been credited to

charge current inhomogeneities<sup>26–30</sup> (which impact the calculations done for non-local setups since they usually assume one dimensional drift-diffusion equations<sup>7,31,32</sup>), or to heating (notably Joule and Peltier heating<sup>33–35</sup>). They may or may not be a nuisance but at any rate they prevent observation of pure non-local voltages. The device we discuss in this paper can be made insensitive to these offset voltages when the two detector electrodes are identical and symmetric since the offsets will cancel out when the voltage difference is measured. This is an additional advantage of our device.

The geometry of our device is that of a Van der Pauw cross as in the Jedema and coll. seminal experiments<sup>4</sup>. A close device within a pure lateral geometry will be discussed elsewhere<sup>36</sup>.

In Section II we introduce the Van der Pauw geometry with four ferromagnetic terminals and give general expressions for the non-local voltage. The basic functionalities of the device are discussed, and notably we will show that the device can perform logic operations (notably as a NOR or NAND gate), be reprogrammed to perform other functions (AND, XOR and inverter gates), displays a potentially interesting ON-OFF switch effect, and when used as a standard 1-bit read-head shows an infinite *GMR* for the non-local resistance. Use as a Direct Spin Hall Effect probe will also be discussed.

The next section III studies in detail the impact of the transparency of interfaces and of the number of ferromagnetic terminals (two or three out of four) on spin confinement, resulting in small or large non-local signals. The signals expected are systematically compared to those in the standard lateral geometry.

The last section IV discusses the main setup with four ferromagnetic symmetric terminals since it displays the previously mentioned properties of (i) immunity to offset voltages and (ii) infinite *GMR* ratio for the non-local resistance. Issues pertaining to the use as a sensor are briefly touched upon.

The bulk of calculations are relegated to the Appendices. Appendix B revisits the bipolar spin switch calculations by including spin leakage in the measuring electrodes.

## II. VAN DER PAUW SETUP.

### A. Geometry and notations.

#### 1. Geometry.

We consider in this section a four-terminal device in a Van der Pauw geometry (see Fig. 1). The terminals are ferromagnets  $F1 - F4$  positioned as in the Figure 1; we allow for arms of unequal lengths. In sections III-III B we will allow some of these terminals to be paramagnetic through a suitable choice of parameters.

**One-dimensional assumption.** We will assume that width and thickness of all arms are much smaller than their length. Experimentally, current inhomogeneities due to departures from strict one-dimensional flow can arise; however the basic functionalities of our device are for the most part independent of that assumption although quantitative predictions may accordingly lose accuracy.

**Injector electrodes.**  $F1$  injects a charge current which is collected in terminal  $F2$ . We will designate them collectively as injector electrodes; when the need to differentiate them shows up, we will say that  $F1$  is the source or injector electrode while  $F2$  is the drain or collector electrode.

**Detector electrodes.** The detection sub-setup consists in terminals  $F3$  and  $F4$  hooked to a voltmeter (or a potentiometer or an ammeter). The latter will measure the non-local voltage as a function of the magnetization orientations of each terminal.

**Orientation.** We define points  $O(x = 0; z = 0)$  the origin and center of the Van der Pauw cross,  $A(x = 0; z = L_1)$ ,  $B(x = 0; z' = L_2)$ ,  $C(x = L_3; z = 0)$  and  $D(x = 0; z' = L_4)$  where each arm has been for later convenience *oriented away* from  $O$  (axis  $Ox$ ,  $Ox'$ ,  $Oz$  and  $Oz'$ ) following Jedema and coll.<sup>4</sup>.

The four paramagnetic arms are:  $I - IV$  (resp.  $OA$ ,  $OB$ ,  $OC$ ,  $OD$ ).

The charge current flowing through  $F1 - I - II - F2$  is  $I_c$  and flows from top to bottom (is therefore negative relative to arm  $I$ , but positive, relative to arm  $II$ ).

The spin accumulation is defined as:

$$\Delta\mu = \frac{(\mu_{\uparrow} - \mu_{\downarrow})}{2e} \quad (1)$$

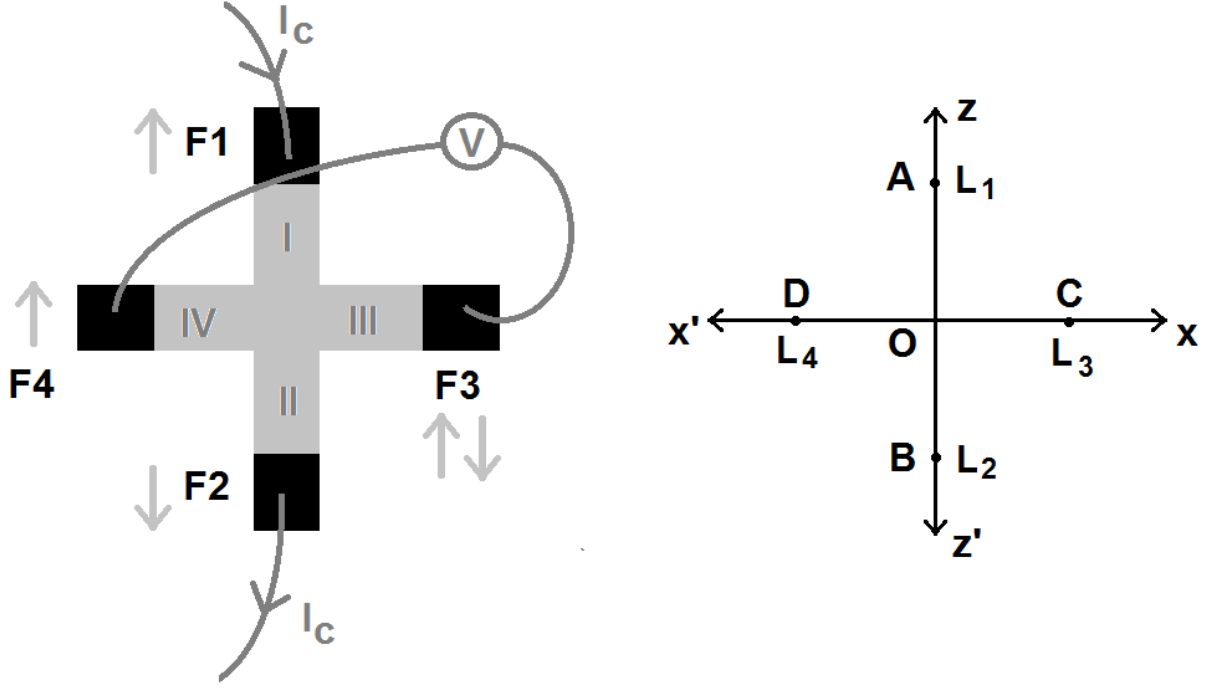


Figure 1. Van der Pauw cross with four ferromagnetic terminals. The arm lengths are respectively  $L_1$ ,  $L_2$ ,  $L_3$  and  $L_4$ . The arrows represent the magnetization direction. When the device is used as a basic spin-valve, only one terminal can switch its magnetization (here  $F3$ ). The arms are oriented *away* from origin  $O$ .

(where for later convenience we have divided by the electron charge  $e$ ).

The spin currents are oriented away from origin  $O$  (therefore are counted positive on a given arm when flowing *away* from  $O$ ).

## 2. Spin parameters.

**Arms parameters.** The central cross is a normal metal. Its parameters are its *spin resistance*  $R_N = \rho_N^* l_N / A_N$  where  $l_N$  is the spin diffusion length,  $A_N$  is the cross-section and  $\rho_N^*$  is the resistivity. We define the lengths of each arm relative to the spin diffusion length as (for  $i = 1 - 4$ ):

$$l_i = \frac{L_i}{l_N}. \quad (2)$$

**Ferromagnet parameters.** For each ferromagnetic terminal  $F1 - F4$  ( $i = 1 - 4$ ), one defines the conductivity polarization  $P_{F,i}$  ( $= \beta_i$  in Valet-Fert notation<sup>31</sup>), *spin resistance*

$R_{F,i} = \rho_{F,i}^* l_{F,i} / A_{F,i}$  where  $l_{F,i}$  is the spin diffusion length,  $A_{F,i}$  is the cross-section and  $\rho_{F,i}^* = (\rho_{i\uparrow} + \rho_{i\downarrow}) / 4$ .

(NB: note on terminology; we will call throughout the paper 'spin resistance' the characteristic resistance found as the product of the resistivity times the spin diffusion length divided by the cross-section ).

**Interface parameters.** At the interface between the ferromagnets and the paramagnetic arms we assume there is a spin dependent interface resistance so that one can define for each interface  $F_i - N$  ( $i = 1 - 4$ ) a conductance polarization  $P_{ci}$  ( $= \gamma_i$  in Valet-Fert notation), a *spin resistance*  $R_{ci} = (R_{i\uparrow} + R_{i\downarrow}) / 4$ . For simplicity we will neglect all spin flips at interfaces so that spin relaxation occurs solely in the bulk of the device.

### 3. Spin resistance mismatch.

We define spin resistance mismatch parameters at F - N interfaces as:

$$X = \frac{R_F + R_c}{R_N}. \quad (3)$$

Three limits can be singled out:

$X < 1$ : this corresponds to the limit of a transparent junction, which as we will see later in detail (Appendix A 2 e) is very leaky in terms of spin: this favors large spin currents at the cost of reduced spin accumulations in the central paramagnet.

$X > 1$ : spin confining or tunneling regime, for which spin accumulation increases but spin current decreases (in magnitude) (see Appendix A 2 e). For  $X \sim 10$ , for which the contact resistance is moderate (about  $10 \Omega$ ) we will say that we are in the weak tunneling limit. This is the most interesting limit in terms of applications to all-metallic read-heads or sensors. For  $R_c = 10^2 - 10^4 \Omega$  which are usual values in tunnel junctions,  $X \sim 10^2 - 10^4$  which we will qualify as strong tunneling limit. Although the strong tunneling regime can be described by our equations, we will focus primarily in the discussions on the transparent and weak tunneling regime where resistances are in the metallic range which interests us for sensor applications.

$X = 1$ : spin impedance matching. The naming for this border situation will be justified below in the discussion on effective spin resistance (section II A 5).

#### 4. *Effective polarizations.*

The following definition will also prove useful. We define for each terminal ( $i = 1 - 4$ ) an effective polarization as:

$$P_{eff,i} = \frac{\widetilde{PR}_i}{\delta_i^+} \quad (4)$$

where:

$$\widetilde{PR}_i = (P_{Fi}R_{Fi} + P_{ci}R_{ci}) / R_N \quad (5)$$

and:

$$\delta_i^\pm = \frac{(X_i + 1)}{2} \exp l_i \pm \frac{(X_i - 1)}{2} \exp -l_i. \quad (6)$$

The effective polarization can be rewritten as:

$$P_{eff,i} = \frac{P_{Fi}R_{Fi} + P_{ci}R_{ci}}{R_N \sinh l_i + (R_{Fi} + R_{ci}) \cosh l_i}; \quad (7)$$

clearly,  $|P_{eff,i}| \leq 1$ .

Upon magnetization reversal of the electrode, the effective polarization is an odd function:

$$P_{eff,i} \longrightarrow -P_{eff,i}.$$

In the limit of short arm length  $l_i \ll 1$ :

$$P_{eff,i} \longrightarrow \frac{P_{Fi}R_{Fi} + P_{ci}R_{ci}}{R_{Fi} + R_{ci}} \quad (8)$$

which is a weighted average of the electrode bulk and interface polarizations.

When  $l_i \longrightarrow \infty$  the effective polarization vanishes exponentially which translates the complete spin relaxation in the arm:

$$P_{eff,i} = 2 \frac{P_{Fi}R_{Fi} + P_{ci}R_{ci}}{R_N + (R_{Fi} + R_{ci})} \exp -l_i. \quad (9)$$

The effective polarization therefore varies between 0 and its maximum value  $\frac{P_{Fi}R_{Fi} + P_{ci}R_{ci}}{R_{Fi} + R_{ci}}$  which is bounded from above by  $\sup(P_{Fi}; P_{ci})$ .

#### 5. *Effective spin resistances.*

We also define an effective spin resistance for each arm of length  $l_i$  ( $i = 1 - 4$ ) as:



$$R_{eff,i}(l_i, X_i) = R_N \frac{\delta_i^+}{\delta_i^-} \quad (10)$$

$$= R_N \frac{X_i \cosh l_i + \sinh l_i}{X_i \sinh l_i + \cosh l_i}; \quad (11)$$

$R_{eff,i}(X_i)$  is an increasing function of spin resistance mismatch  $X_i$ .

It is shown in Appendix A 2 b that the spin current  $I_{s,i}(O)$  at the cross center  $O$  on arm  $i$  and the spin accumulation there are related by  $\Delta\mu(O) = -R_{eff,i} I_{s,i}(O)$  (for arms  $III - IV$ ; for arms  $I - II$  a more general relation taking into account the spin injection at  $F1$  and  $F2$  holds). This is an analog of Ohm's law for spin which explains our identification of  $R_{eff,i}$  as a spin resistance. (Note that the analogy is not complete: the relation for spin is a local one (expressed here at point  $O$ ), while Ohm's law holds for a voltage difference and is therefore non-local. This results of course from the non-conservation of spin current.)

We also define a total effective spin resistance for the device:

$$R_{eff} = \frac{1}{\sum_{i=1-4} \frac{1}{R_{eff,i}}}. \quad (12)$$

The previous expression admits obvious generalization to an arbitrary number  $n \geq 4$  of arms.

As can be seen from its definition, the total effective resistance  $R_{eff}$  is related to the arms spin resistances  $R_{eff,i}$  by the analog of a parallel resistance addition law. We will show later (section II B 2) that the spin voltage is proportional to the total effective spin resistance  $R_{eff}$  so that large effective resistances are desirable; this will also effectively demonstrate for our geometry the parallel addition law for spin resistances. (For a discussion of spin resistance addition law at nodes we refer the reader to<sup>37</sup>.) Since the total effective spin resistance is the sum of four resistances in parallel, whenever one is much smaller than the others, it will short the other arms: spin leakage will be stronger so that spin accumulation will be reduced.

The length dependence of the effective resistance for various values of the spin resistance mismatch  $X$  is shown in Fig. 2-3 . At large distance the effective resistance converges exponentially fast to  $R_n$  the spin resistance of the paramagnetic arm:

$$R_{eff,i} \longrightarrow R_N \quad (13)$$

which reflects the fact that the spin relaxation is dominated by the paramagnet bulk. In the limit  $l_i \rightarrow 0$

$$R_{eff,i} \rightarrow R_{Fi} + R_{ci} \quad (14)$$

(which is sensible since the paramagnet is then too short for spin relaxation to occur).

When  $l_i \ll 1$ , the effective resistance remains close to  $R_{Fi} + R_{ci}$  if the distance  $l$  obeys:

$$l |X^2 - 1| < X.$$

When  $X \gg 1$ , the condition becomes  $l \ll 1/X$  which reflects a steeper exponential decrease.

The effective spin resistance for a given arm is therefore comprised between  $R_N$  and  $R_{Fi} + R_{ci}$ :

$$R_N \leq R_{eff,i} \leq R_{Fi} + R_{ci} \quad (15)$$

(or the reverse inequality if  $R_N \geq R_{Fi} + R_{ci}$ ).

As a rule the effective spin resistance will be larger for large interface or ferromagnet spin resistance ( $R_c$  and  $R_F$ ); since the ferromagnet spin resistance  $R_F$  is in general much smaller than the paramagnet spin resistance  $R_N$  due to short spin diffusion lengths, large interface resistances  $R_c$  are required to achieve large effective resistances  $R_{eff,i}$ .

For  $X = 1$ , one observes that the effective spin resistance does not depend any more on the arm length  $l_i$  and is equal to  $R_N$ . One then has (on arms *III* or *IV*)  $\Delta\mu(O) = -R_N I_{s,i}(O)$  which is the relation one would get from an infinite arm. Everything happens as if the interface had been washed away: this is the reason why we qualified the case  $X = 1$  as corresponding to spin impedance matching in II A 3.

In the transparent regime ( $X < 1$ ), the effective spin resistance is larger at large distance, which is an interesting feature for the design of large non-local circuits. This advantage is circumvented by the exponential decrease of the effective polarization (the non-local resistance will be shown to be proportional to both in II B 2) so that in terms of large signals the transparent regime is not interesting, neither in the short-distance nor the large-distance limit.

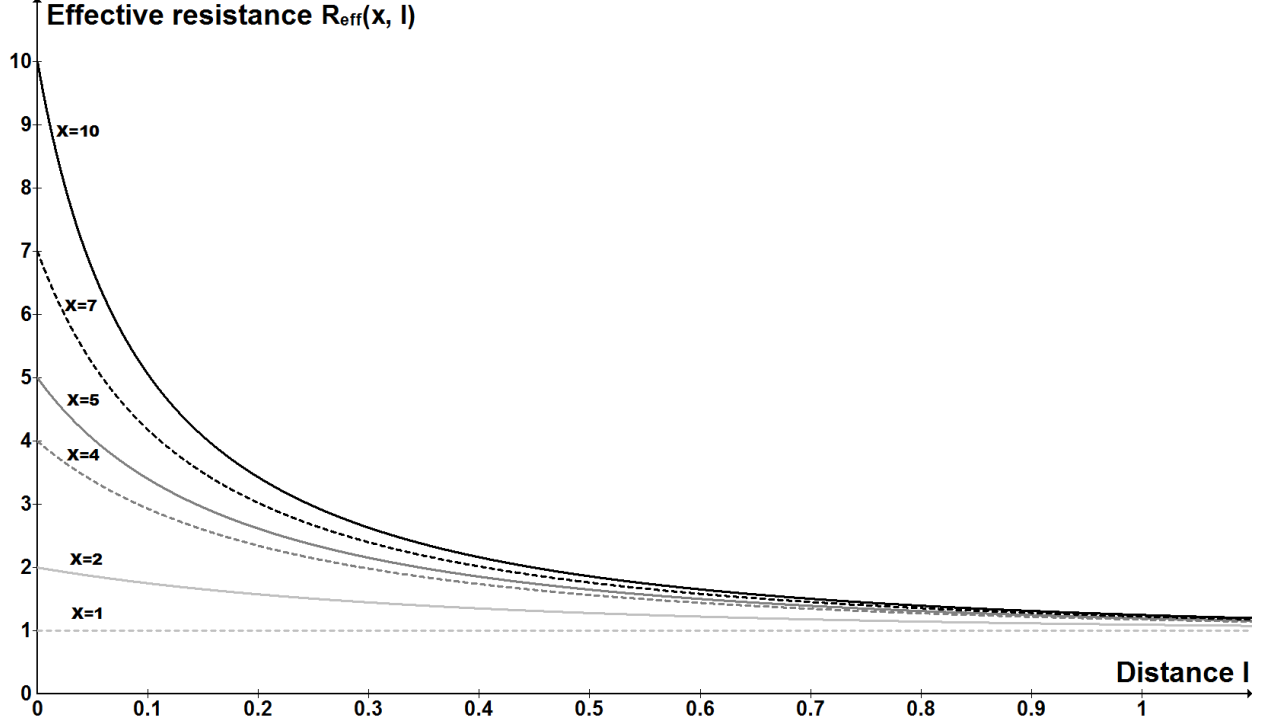


Figure 2. Effective resistance for one terminal (normalized to  $R_N$ ) as a function of arms length  $l$  (relative to paramagnet spin relaxation length) for various impedance mismatches  $X = 1 - 10$  in the (weak) tunneling regime.

## B. General expression of the spin voltage in Van der Pauw geometry.

### 1. Spin accumulation at cross center.

Solving the one-dimensional drift-diffusion equations (see Appendix A for details of the calculations) leads to the following results.

The spin accumulation in the center of the cross is found as:

$$\Delta\mu(O) = (P_{eff,1} - P_{eff,2}) R_{eff} I_c \quad (16)$$

We can define a total effective polarization at injector electrodes  $F1$  and  $F2$ :

$$P_{injector} = P_{eff,1} - P_{eff,2}. \quad (17)$$

The minus sign in front of  $P_{eff,2}$  comes from the fact that  $F2$  is a charge drain: in terms of spin accumulation it therefore acts contrariwise to electrode  $F1$ . This also shows that to ensure maximum signal it is better to have opposite orientations for  $F1$  and  $F2$  (antiparallel

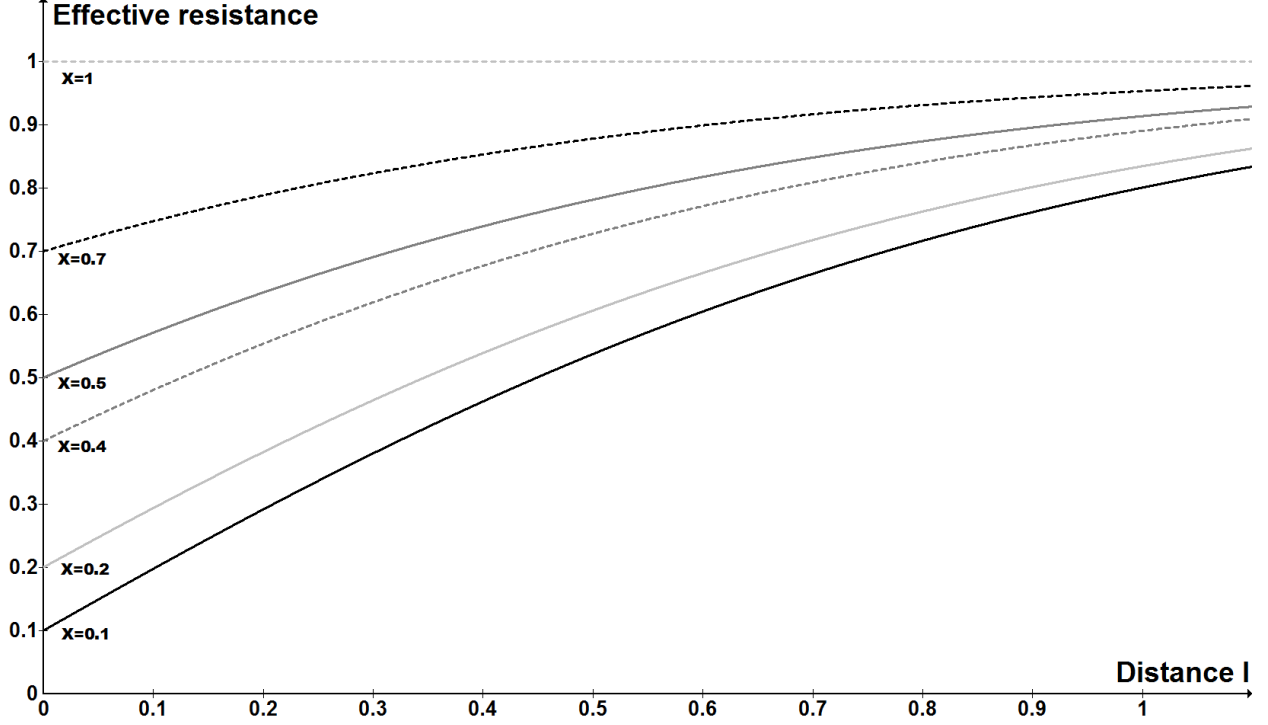


Figure 3. Effective resistance for one terminal (normalized to  $R_N$ ) as a function of arms length  $l$  (relative to paramagnet spin relaxation length) for various impedance mismatches  $X = 0.1 - 1$  in the transparent regime ( $X < 1$ ).

injector electrodes). This is easy to understand: when injector electrodes are parallel,  $F2$  acts as a spin sink for the spins injected by  $F1$ ; therefore the spin accumulation should decrease. But when  $F2$  is antiparallel to  $F1$ , spin leakage is frustrated; although  $F2$  is a charge drain, it acts as an additional spin source.

It is noteworthy that the spin accumulation does not depend on the magnetization orientations of detector electrodes  $F3$  and  $F4$  (although it does depend on its parameters through  $R_{eff}$ ). This is sensible: spin injection is ensured by  $F1$  and  $F2$  not  $F3$  and  $F4$ .

The general structure of the spin accumulation in terms of  $F1$  and  $F2$  relative orientation can be understood simply. Suppose we flip all spins of the setup. Then:  $\Delta\mu \rightarrow -\Delta\mu$  since we have exchanged spin up and spin down electrons. This implies that the spin accumulation must be an odd function of electrode polarizations. This is easily checked on Eq. (16): when both  $F1$  and  $F2$  are flipped, their effective polarizations get reversed  $P_{eff} \rightarrow -P_{eff}$  and therefore the spin accumulation is reversed. Another way to reach the same conclusion is to notice that the physics of the setup should be invariant when both

current and magnetizations are reversed. Flipping all spins or reversing the charge current should lead to the same spin accumulation, namely a reversed one.

An important consequence of the structure of Eq. (16) is that the spin accumulation at the cross center vanishes when  $F1$  and  $F2$  are identical and parallel so that  $P_{eff,1} = P_{eff,2}$ . This follows clearly from symmetry: when  $F1$  and  $F2$  are symmetric with respect to the cross center and have identical parameters, the spin accumulation on the line  $F1-F2$  should be antisymmetric and the spin current should be symmetric when  $F1$  and  $F2$  are parallel (the opposite when they are antiparallel). This can be understood by reversing the current: if one reverses the current, on the one hand, the spin accumulation at the cross center should not change since  $F1$  and  $F2$  are identical and parallel (by watching Fig. 1 after a  $\pi$  rotation); on the other hand, reversing the current must reverse the spin accumulation since reversing the current is equivalent to reversing all spins. The only way out is for the spin accumulation at the cross center to vanish. This result does not depend on the assumption of one-dimensional flow and follows directly from symmetry.

We will see later that this enables an ON-OFF switch function onto the device.

## 2. Spin voltage and non-local resistance.

**Spin voltage.** The spin voltage (or non-local voltage) is the voltage drop between terminals  $F3$  and  $F4$ :

$$V_{nl} = -[\mu_{F3}(+\infty) - \mu_{F4}(+\infty)]/e. \quad (18)$$

Straightforward calculations (see Appendix A 2 d) lead to:

$$V_{nl} = -\Delta\mu(O) (P_{eff,3} - P_{eff,4}). \quad (19)$$

The behaviour under magnetization reversal is easy to understand: when the magnetizations of both detector electrodes are switched, their coupling to the spin accumulation gets reversed so that the non-local spin voltage should change sign. The spin voltage is therefore odd under magnetization switching of both detector electrodes.

Inserting the expression of the spin accumulation in Eq. (16):

$$V_{nl} = (P_{eff,1} - P_{eff,2}) (P_{eff,3} - P_{eff,4}) R_{eff} I_c \quad (20)$$

This expression factors out neatly in three contributions:

- (i) the geometry dependent effective spin resistance  $R_{eff} = (\sum_{i=1-4} R_{eff,i}^{-1})^{-1}$ ;
- (ii) a total effective polarization for injector and collector electrodes ( $F1$  and  $F2$ )

$$P_{injector} = P_{eff,1} - P_{eff,2}; \quad (21)$$

- (iii) and a total effective polarization for the two detector electrodes ( $F3$  and  $F4$ ):

$$P_{detector} = P_{eff,3} - P_{eff,4}. \quad (22)$$

**Non-local resistance.** The non-local resistance (sometimes called a transresistance) is defined as the ratio of the non-local voltage to the charge current flowing through the injector electrode  $F1$  to the collector electrode  $F2$  :

$$R_{nl} = \frac{V_{nl}}{I_c} = R_{eff} P_{eff,injector} P_{eff,detector}. \quad (23)$$

To achieve a large signal it is therefore necessary to have a large total effective spin resistance for the cross  $R_{eff}$  and to have on the one hand antiparallel source and drain terminals, on the other hand antiparallel detector electrodes.

$R_{eff}$  is largest when all arms effective spin resistances are also large, which is the case if spin resistance mismatches are in the tunneling regime according to the discussion in II A 5 and if the arms length is short enough. Two situations may arise:

i) one or several spin resistance mismatches are in the transparent regime ( $X \leq 1$ ). Then  $R_{eff} \sim R_N$  or smaller. No enhancement of the spin voltage is to be expected when compared with the usual lateral setup. We will say that we have an open geometry<sup>23</sup> which leaks spin (larger spin currents but smaller spin accumulations).

ii) all spin resistance mismatches are in the spin confining regime ( $X > 1$ ). Then  $R_{eff} \sim R_c$  in the limit of short length for the arms (and assuming for simplicity mismatches roughly equal  $X_i \sim X \sim R_c/R_N$ ). In such a geometry which will be qualified as closed<sup>23</sup> the signal is therefore potentially much larger than in an open geometry.

Let us compare the non-local resistance to local resistances in the same device.

i) Firstly a local resistance can be measured between source and drain ( $F1$  and  $F2$ ); as shown in Appendix A 3 a:

$$R_{local,12} = R_{eff} [P_{eff,1} - P_{eff,2}]^2 + R_0 \{P_{1-2}\} \quad (24)$$

where

$$R_0 \{P_{1-2}\} = \sum_{i=1-2} \rho_{Fi}^* (1 - P_{Fi}^2) z_i + \rho_N^* l_i + R_{ci} + \frac{[P_{Fi}^2 R_{ci} R_{Fi} - P_{ci}^2 R_{ci}^2 - 2P_{Fi} P_{ci} R_{Fi} R_{ci}]}{R_{Fi} + R_{ci}}. \quad (25)$$

( $z_i$  are the locations of probes in the electrodes, see Appendix A 3 a). In general  $R_{local,12} \approx R_0$  since the GMR effect is a few percents; the non-local resistance which is commensurate with  $R_{local,12} - R_0$  is therefore much smaller than  $R_{local,12}$ .

It is more meaningful to compare the variations upon magnetization switching  $\Delta R = R_{AP} - R_P$ ; for the non-local signal, we have set in the following  $F1$  and  $F2$  antiparallel while switching  $F4$  magnetization:

$$\frac{\Delta R_{nl}}{\Delta R_{local,12}} = \frac{(P_{eff,1} + P_{eff,2}) P_{eff,4}}{2P_{eff,1} P_{eff,2}} \quad (26)$$

(where for  $R_{local}$ , either terminal  $F1$  or  $F2$  have been switched). In the case of identical source and drain electrodes ( $F1$  and  $F2$ ) this reduces to:

$$\frac{\Delta R_{nl}}{\Delta R_{local,12}} = \frac{P_{eff,4}}{P_{eff,1}}, \quad (27)$$

if the ratio is larger than unity ( $P_{eff,4} > P_{eff,1}$ ) there is an amplification of non-local MR variation versus local MR (or the converse if  $P_{eff,4} < P_{eff,1}$ ).

ii) If we then compare to the local resistance found when current flows from  $F3$  to  $F4$ , one gets instead:

$$\frac{\Delta R_{nl}}{\Delta R_{local,34}} = \frac{(P_{eff,1} + P_{eff,2})}{2P_{eff,3}}, \quad (28)$$

One can again get an MR amplification (or reduction if  $P_{eff,3}$  is sufficiently large).

But if all electrodes are identical, all these MR ratios are then equal to unity

$$\frac{\Delta R_{nl}}{\Delta R_{local}} = 1 \quad (29)$$

which means the non-local measurement performs equally well as local measurements in terms of raw resistance variation, with one proviso: non-local resistances have smaller baselines. Indeed for  $R_{nl}$ , the baseline or smallest signal is:

$$|R_{nl,min}| = R_{eff} (|P_{eff,1}| - |P_{eff,2}|) (|P_{eff,3}| - |P_{eff,4}|)$$

while for  $R_{local,12}$ :

$$|R_{local,min}| = R_0 \{P_{1-2}\} + R_{eff} (|P_{eff,1}| - |P_{eff,2}|)^2$$

which is larger on account of the  $R_0$  term.

iii) It is worthwhile to compare to CPP GMR for a spin valve with similar dimensions:

$$\Delta R_{CPP} = \frac{4R_N (P_c R_c + P_F R_F)^2}{2R_N (R_F + R_c) \cosh l + [(R_F + R_c)^2 + R_N^2] \sinh l}$$

for identical and infinite ferromagnetic layers separated by the same distance  $L = l l_N$ . In terms of  $X = (R_F + R_c) / R_N$  this can be recast as:

$$\Delta R_{CPP} = \frac{8R_N^{-1} (P_c R_c + P_F R_F)^2}{(X + 1)^2 \exp l - (X - 1)^2 \exp -l}$$

so that for a non-local device with identical terminals:

$$\frac{\Delta R_{nl}}{\Delta R_{CPP}} = \frac{1}{2}. \quad (30)$$

The non-local signal is smaller by a factor 2 which is easy to understand: in the cross geometry the spin relaxation volume is doubled when compared with a spin valve with a paramagnetic layer of identical length because of the side arms.

A systematic comparison of the non-local resistance in the cross geometry with the standard lateral geometry is left to sections III A-IV.

### 3. Non-local charge current.

The voltage between the detector electrodes  $F3$  and  $F4$  actually acts as an electromotive force (emf) of magnetic origin; when  $F3$  and  $F4$  are shorted, a charge current therefore appears. This non-local charge current is induced by the spin accumulation generated by the remote source and drain  $F1$  and  $F2$ . Note that since the spin voltage is an electromotive force, it can be measured either through a (nano-)voltmeter or with a potentiometer: in the latter case, the advantage is that there is no current at all during the measurement.

If however it proves advantageous that the signal be a current (for chaining the non-local device to a bipolar transistor for instance rather than a MOSFET), the non-local current is found (see Appendix A 3 b) as:

$$I_{nl} = -\frac{R_{nl}}{R_{local,34}} I_c \quad (31)$$

where  $R_{local,34}$  is the local resistance measured when one drives a current between  $F3$  and  $F4$ . This can be rewritten as:

$$I_{nl} = -\frac{R_{eff} (P_{eff,1} - P_{eff,2}) (P_{eff,3} - P_{eff,4})}{R_{eff} [P_{eff,3} - P_{eff,4}]^2 + R_0 \{P_{3-4}\}} I_c \quad (32)$$



where:

$$R_0 \{P_{3-4}\} = \sum_{i=3-4} \rho_{Fi}^* (1 - P_{Fi}^2) z_i + \rho_N^* l_i + R_{ci} + \frac{[P_{Fi}^2 R_{ci} R_{Fi} - P_{ci}^2 R_{ci}^2 - 2P_{Fi} P_{ci} R_{Fi} R_{ci}]}{R_{Fi} + R_{ci}}. \quad (33)$$

(it can be checked that  $R_0 \geq 0$ );  $z_i$  ( $i = 3 - 4$ ) are the locations on  $F3$  and  $F4$  of the wires which short them together (in the following we have chosen to focus on  $R_{nl}$  rather than  $I_{nl}$ ).

### C. Main properties of the device.

The device functionalities depend on the terminals for which the magnetization has been fixed (prevented from switching); additional symmetry requirements can also add properties. We first discuss functionalities pertaining to sensing or data storage.

As explained in the introduction the non-local resistance and spin voltage are contaminated by offsets, the origin of which is still under debate (current inhomogeneities; thermal origin). This may adversely affect the measured signals and in the following we will take care to indicate the potential impact of these offsets.

**1-bit reading.** This is the basic functionality of the spin valve as a sensor. Suppose the orientations of all terminals but one (say  $F3$ ) are pinned (for definiteness we assume:  $F1 : \uparrow$ ,  $F2 : \downarrow$  and  $F4 : \downarrow$ ) so that  $F3$  acts as a sensing electrode. We make no special assumptions on the terminals (later on some conditions will be imposed for further functionalities). One recovers a spin-valve behaviour (see Fig. 4) with two distinct values of the non-local resistance which uniquely determine the orientation of terminal  $F3$ :

$$R_P = R_{eff} (P_{eff,1} + P_{eff,2}) (P_{eff,3} - P_{eff,4})$$

$$R_{AP} = R_{eff} (P_{eff,1} + P_{eff,2}) (P_{eff,3} + P_{eff,4})$$

(the  $P/AP$  index refer to  $F3$  magnetization orientation relative to  $F4$ ). The spin valve can therefore be used to read a 1-bit information.

In the standard non-local spin valve and if we neglect voltage offsets, the spin voltage changes sign when one terminal is flipped so that for antiparallel and parallel alignment  $R_{AP} = -R_P$  (this is recovered here in the limit  $P_{eff,4} = 0$ , when  $F4$  is a paramagnet); this is not the case here although depending on the relative values of  $P_{eff,3}$  and  $P_{eff,4}$  there can still be a change of sign (Fig. 4-b, when  $P_{eff,3} > P_{eff,4}$ ).

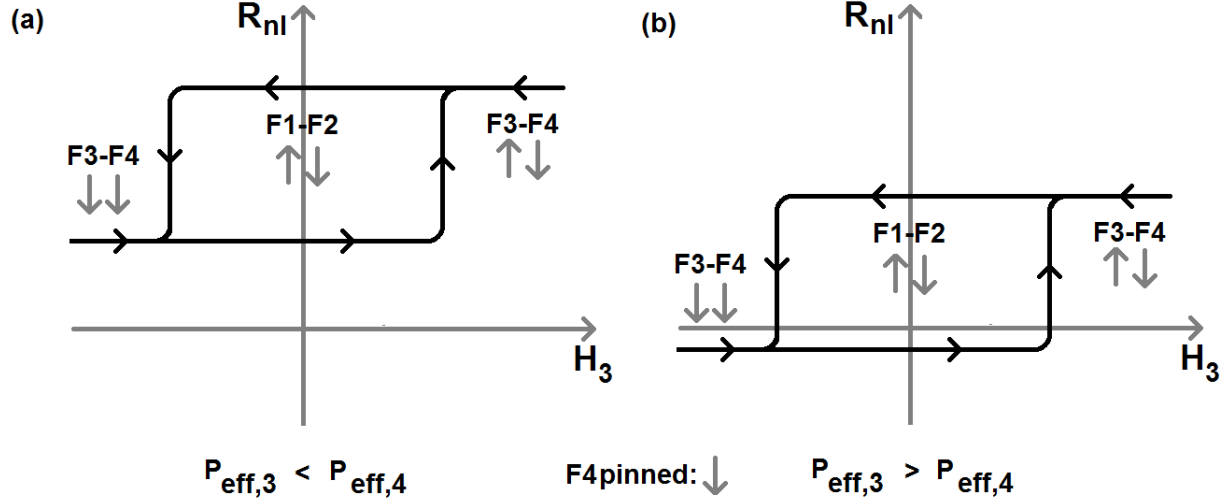


Figure 4. Non-local resistance variation when the orientation of a single terminal (here  $F3$ ) is switched by an external magnetic field (here  $H_3$ ). All other terminals are pinned. Injector  $F1$  and collector  $F2$  are antiparallel with orientation as shown on the graph. (a) case  $P_{eff,3} < P_{eff,4}$ : standard spin valve effect (1-bit reading); (b) case  $P_{eff,3} > P_{eff,4}$ : spin valve effect with spin voltage change of sign.).

For the Van der Pauw cross under the most general conditions the difference with the standard non-local setup is therefore minor; it remains to see if larger signals can be achieved. This will be the topic of sections III-IV where we will show that tunnel contacts at the four terminals can greatly enhance the non-local resistance.

Furthermore this functionality is clearly affected by offset voltages which will shift the signals and change the GMR ratios (adversely if the offset is positive). We now discuss a simple way to circumvent these offsets.

### Offset free 1-bit reading with infinite non-local GMR.

The offset issue is easily fixed if we assume  $F3$  and  $F4$  are identical electrodes placed symmetrically with respect to the rest of the setup. Due to symmetry offset voltages are neutralized (assuming offset voltages are spin independent) since they will shift both voltages  $V_3$  and  $V_4$  in the same manner so that the spin voltage  $V_{nl} = V_3 - V_4$  is free from offsets.

Additionally:  $R_P = R_{eff} (P_{eff,1} + P_{eff,2}) (P_{eff,3} - P_{eff,4}) = 0$  in parallel alignment

since  $P_{eff,3} = P_{eff,4}$  while  $R_{nl} \neq 0$  in antiparallel alignment (we have assumed that  $F1$  and  $F2$  are antiparallel and pinned).

One has achieved an infinite GMR for the non-local resistance since the ratio  $GMR = (R_{AP} - R_P)/R_P \rightarrow \infty$  (using the optimistic ratio; the pessimistic ratio would be 100 %). There is an intrinsic contrast which is protected by symmetry from the voltage offsets.

Such a maximized MR ratio is clearly helpful for SNR in terms of Johnson noise (or shot noise in the strong tunneling limit) since the latter scales as  $\Delta R/\sqrt{R}$ ; indeed if we compare with a CPP spin valve with a  $\Delta R/R = 1 - 10$  % GMR ratio, this would imply at identical  $\Delta R$  ( $\Delta R = \Delta R_{local} = \Delta R_{nl}$ ) an increase of SNR by 10 – 20 dB. (Indeed: SNR for non-local device would be  $\propto 1/\sqrt{\Delta R}$  so that  $SNR_{non-local}/SNR_{CPP} \propto \sqrt{R_{local}/\Delta R}$ ). The difference is quite significant given that under operation one expects in general at least 30 dB SNR.

In terms of geometry requirements, note that the symmetry between detector electrodes is required only on the scale of a few spin relaxation lengths  $l_F$  (on the ferromagnet side) since the spin accumulation is washed at larger distance.

The property is also clearly independent of the precise geometric arrangement, does not depend on the assumption of one-dimensional flow and will be valid for other geometries than the cross studied in this paper, provided the two detectors are arranged symmetrically with respect to the injectors. Experimentally this is very useful since this gives a lot of flexibility in terms of design.

**3-bit reading or storage.** For that function one terminal is pinned, while the other three are free and play the role of input signals (the non-local resistance measured between  $F3$  and  $F4$  is as previously the output signal). Eq. (23) shows the non-local resistance can assume 8 different values when the electrodes orientation are changed and the maximum value for  $|R_{nl}|$  is reached when on the one hand  $F1$  and  $F2$  are antiparallel, and on the other hand  $F3$  and  $F4$  are also antiparallel. The orientations of the three non-pinned terminals are uniquely determined by the 8 distinct values of the spin voltage. This implies that the device encodes 3 bits in principle (3 bit spin valve).

Fig. 5 shows the eight outputs signals as a function of the input variables (for illustrative purposes  $F3$  is varied by an external field  $H3$  on the graphs, showing the output variation with the change of one input).

The property survives offset voltages which come as an additive contribution to the voltage. However resolution may be adversely affected by offsets. Note that although there

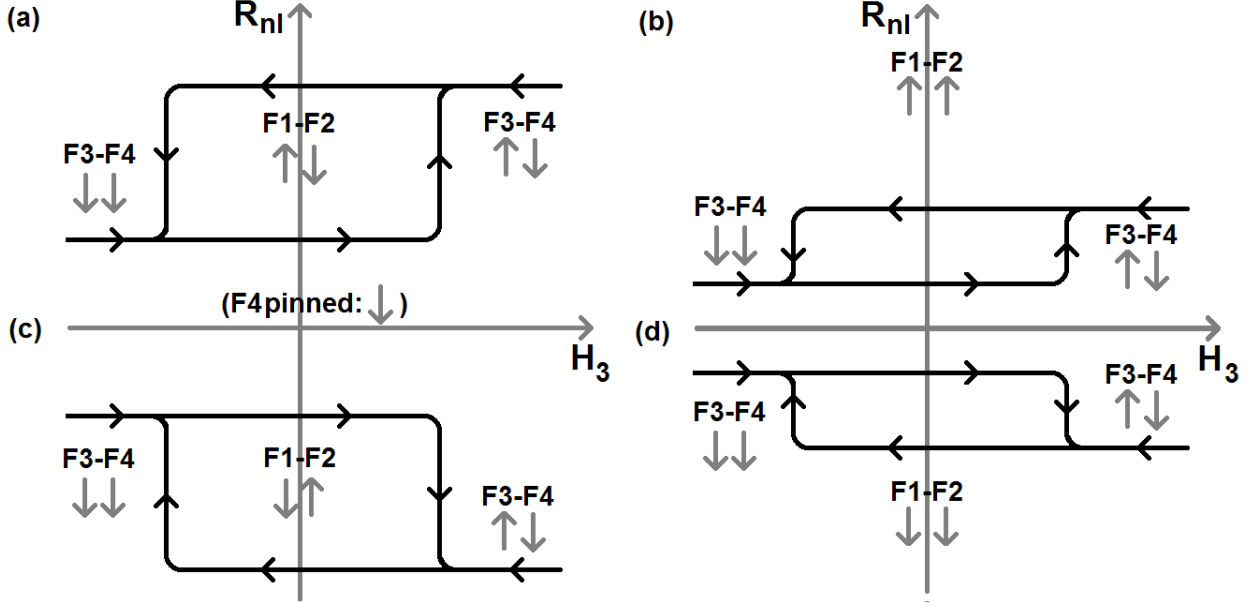


Figure 5. Non-local resistance variation as a function of terminals magnetization orientation showing 3-bit sensor or storage. For illustrative purposes  $F3$  magnetization is varied in each figure (a-d) (through an applied field  $H3$ ) to show the output signal variation with the change of one input. (a) Injector  $F1$  and collector  $F2$  are antiparallel. (b) They are switched to parallel: the signal collapses (to some extent). (c) Signal is reversed when injector and collector are both switched from antiparallel alignment in (a). (d) Injector and Collector are parallel (but with directions opposite to (b) ).

are four ferromagnets and a priori 4 bits could be stored, the spin voltage can assume only 8 values, not  $2^4 = 16$ . This is because when all spins are reversed, the spin voltage is unchanged in Eq. (23).

**Offset-free 2-bit reading.** By the same token applied previously to detectors for the 1-bit reading it is possible to convert the 3-bit reading function into a protected 2-bit reading function: suppose that detector terminals  $F3$  and  $F4$  are symmetric (namely, have identical parameters and are placed symmetrically with respect to the device) but that  $F1$  and  $F2$  differ ( $P_{eff,1} \neq P_{eff,2}$ ). Let us pin terminals 3 and 4 in antiparallel orientation ( $F3 : \uparrow$  and  $F4 : \downarrow$ ), while only  $F1$  and  $F2$  are allowed to switch magnetizations. The non-local spin voltage then takes 4 distinct values depending on the orientations of terminals 1 and 2:

$R_{nl} = R_{eff} (P_{eff,3} + P_{eff,4}) (\pm P_{eff,1} \pm P_{eff,2})$ . These values uniquely determine the 2-bit state of  $F1$  and  $F2$ . The main advantage when compared with the 3-bit function discussed previously is that since terminals  $F3$  and  $F4$  are symmetric, there can be no offset voltages since they automatically cancel out when measuring the voltage drop between  $F3$  and  $F4$  (if offset voltages are spin-independent; if it is not the case, they will add up to the spin voltage). Note that if  $F1$  and  $F2$  are pinned while  $F3$  and  $F4$  are not pinned but identical (to avoid offsets), the spin voltage only assumes three values since  $R_P = 0$ ; to store 2 bits without offsets it is therefore necessary to pin  $F3$  and  $F4$ , not the other way around.

**ON-OFF switch for the 1-bit read-out or storage function.** When source  $F1$  and drain  $F2$  are identical ferromagnets (same distance from origin, same conductivities, polarizations, interface resistances, spin diffusion length),  $P_{eff,1} = P_{eff,2}$ . This then implies that when  $F1$  and  $F2$  are parallel, the spin accumulation at the cross center vanishes

$$\Delta\mu(O) = 0 \quad (34)$$

so that whatever the orientation of  $F3$  and  $F4$ ,  $R_P = 0 = R_{AP}$ . The spin voltage has been killed and we have disabled the read-head or 1-bit storage. The property is clearly interesting in terms of logic if the device is chained to another device for instance a MOSFET whose gate is controlled by the spin voltage (after suitable amplification).

That property should survive offset voltages since symmetry protects it.

But one might wonder if the spin voltage measured at  $F3$  and  $F4$  will still vanish if we take into account departures from strict one-dimensional charge and spin flow. It is clear indeed that even if we take into account the 3-dimensional nature of the device but remain in a quasi-one-dimensional approximation, the spin accumulation on the side arms (zones  $III - IV$ ) will remain small (it may be non zero at edges) and will never diffuse as far as  $F3$  and  $F4$  provided the width of each arm is much smaller than its length ( $w \ll l$ ): indeed when  $F1$  and  $F2$  are parallel, the spin accumulation is an odd function of  $z$  in the direction  $F1 - F2$ ; therefore, on the side arms ( $III - IV$ ), there will be some spilling of spin accumulation with opposite signs on opposite edges, close to the origin  $O$ . But if  $w \ll l$ , spin diffusion will mix these opposite spin accumulations which will cancel out when one reaches the detector terminals ( $F3$  and  $F4$ ). The spin voltage measured at  $F3$  and  $F4$  should therefore still vanish.

**Direct Spin Hall Effect probe.** The geometry lends itself easily to probing the Spin

Hall Effect<sup>6,38,39</sup>. Imagine  $F1$  and  $F2$  are normal electrodes and that the magnetizations of  $F3$  and  $F4$  are perpendicular to the plane of the cross. On the arm  $AB$  ( $F1-I-O-II-F2$  or  $zOz'$ ) a spin accumulation may appear due to Hall coupling  $\alpha_H = \sigma_{xx}/\sigma_{xy}$  on the width  $w$  (for  $-w/2 \leq x \leq w/2$ ):

$$\Delta\mu(x, z) = \frac{\alpha_H \Delta V}{L_1 + L_2} x. \quad (35)$$

When we reach the cross center, there can be a spilling of charge current lines to the side arms, but we will neglect this effect by assuming that the arms width are much smaller than their lengths ( $w \ll L_i$  for  $i = 1 - 4$ ). When we move on the side arms ( $III - IV$ ), the spin accumulation decreases in magnitude. This decrease can be estimated using drift-diffusion equations as:

$$\Delta\mu_{III}(C) = \frac{X_3}{\delta_3^+} \Delta\mu(x = \frac{w}{2}, z = 0) \quad (36)$$

[see Appendix A 2 d, Eq. (A48)] with a similar expression for arm  $IV$ :

$$\Delta\mu_{IV}(D) = \frac{X_4}{\delta_4^+} \Delta\mu(x = -\frac{w}{2}, z = 0) \quad (37)$$

$\Delta\mu_{III}(C)$  can be re-expressed as:

$$\Delta\mu_{III}(C) = \frac{X_3}{\delta_3^+} \frac{\alpha_H \Delta V}{L_1 + L_2} \frac{w}{2} \quad (38)$$

and:

$$\Delta\mu_{IV}(D) = -\frac{X_4}{\delta_4^+} \frac{\alpha_H \Delta V}{L_1 + L_2} \frac{w}{2}. \quad (39)$$

The spin voltage is therefore (see Appendix A 2 d):

$$V_{nl} = -(P_{eff,3} + P_{eff,4}) \Delta\mu(x = \frac{w}{2}) \quad (40)$$

$$= -(P_{eff,3} + P_{eff,4}) \frac{\alpha_H \Delta V}{L_1 + L_2} \frac{w}{2}. \quad (41)$$

Note that for identical and antiparallel electrodes  $F3$  and  $F4$ , the signal therefore vanishes since  $P_{eff,3} = -P_{eff,4}$  and that for maximal non-local signal, parallel magnetizations are required.

Let us go back to our initial setup with four ferromagnetic electrodes. If magnetizations are perpendicular to the plane of the Van der Pauw cross, the spin Hall effect will add up to the non-local spin voltage arising from spin injection. But if we choose a symmetric setup with identical parallel electrodes  $F3$  and  $F4$ , offsets cancel out in  $V_{nl} = V_3 - V_4$  and  $V_P = 0$

as explained previously. Therefore the only remaining signal is that of the Direct Spin Hall Effect. This provides an interesting all-electrical alternative to the observation of Direct Spin Hall Effect, which initially was observed optically in semiconductors<sup>40,41</sup> although later on some electrical detection schemes have been used in metallic systems to investigate both Direct and Inverse Spin Hall Effect<sup>42–44</sup>.

#### D. Implementation of magneto-logic gates.

Many proposals exist in the literature for the use of magnetoelectronics circuits as logic gates<sup>45</sup>; this has prompted a lot of activity in the field of semiconductor spintronics since integration to existing processes would be optimal<sup>46</sup>. Our device is metallic and therefore not the best candidate as a spin transistor since there is no amplification: this renders the chaining of gates more delicate for instance (unless one uses hybrid designs combining pure magnetoelectronics devices with conventional transistors). However, the device still possesses obvious capability as a programmable magneto-logic gate as we now demonstrate.

Let us associate bit 0 with down  $\downarrow$  magnetization and bit 1 with up  $\uparrow$  magnetization.  $F1$  and  $F2$  are assumed to be identical;  $F3$  and  $F4$  are also identical (same arm length, same polarizations, spin diffusion length, etc); only the magnetization orientations are allowed to differ. The non-local resistance can then assume only three values: 0 and  $\pm R_0 = \pm 4R_{eff} P_{eff,1} P_{eff,3}$ . For logic operations we will consider two conventions:

- (i) associate bit 0 to zero resistance, and bit 1 to  $R_{nl} = +R_0$ .
- (ii) or (opposite convention) associate bit 1 to zero resistance, and bit 0 to  $R_{nl} = +R_0$ .

Note that in what follows we have discarded all configurations for which  $R_{nl} = -R_0$  to avoid ambiguities in the bit association and keep only those for which there are only two possible outputs: 0 and  $R_0$ .

For the first convention assigning the bit content of  $R_{nl}$ , the expression for  $R_{nl}$  [see Eq. (23)] can therefore be rewritten in terms of bits as the Boolean equation:

$$(F1 - F2)(F3 - F4) = R_{nl} \quad (42)$$

where to simplify notation we have conflated  $R_{nl}$  and its bit content.

With the opposite convention,

$$(F1 - F2)(F3 - F4) = \overline{R_{nl}} \quad (43)$$

Note that in order to have well defined HIGH and LOW states the associated voltages (or non-local resistances) must be sufficiently different which is not ensured in the presence of offset voltages. That's why we choose symmetric terminals to get rid of these offsets.

We first try to reproduce basic binary Boolean functions. We need therefore to pin two terminals; the other two terminals will represent the variables treated by the device in the following manner (for instance): imagine each terminal is screened to prevent any magnetic field applied to a given electrode to influence any other electrode; we then apply external magnetic fields to either of the two non-pinned terminals but not to the other twos, whose magnetizations are therefore fixed during the whole operation.

**NOR gate.** We choose convention of Eq. (42) for bit coding. We pin  $F2$  and  $F4$  ( $F2 : \uparrow$  and  $F4 : \uparrow$ ) so that  $F2 = 1$  and  $F4 = 1$  in bit terms. To have a non-zero resistance among the four possible bit configurations of  $F1$  and  $F3$ , only  $F1 = 0$  and  $F3 = 0$  are allowed. The non-local resistance is then  $R_{nl} = +R_0$  which we associate with HIGH state or bit 1: this means that  $\overline{F1} \overline{F3} = R_{nl}$ ; this can also be recovered through algebra by using Eq. (42) which in our case is:

$$(F1 - 1)(F3 - 1) = R_{nl} \quad (44)$$

so that:

$$\overline{F1} \overline{F3} = R_{nl}. \quad (45)$$

This is precisely a NOR gate since  $\overline{F1} \overline{F3} = \overline{F1 + F3}$  by De Morgan theorem. This is a very important property since NOR has functional completeness: any Boolean function can be implemented by using a combination of NOR gates (only NAND possesses the same property).

**OR gate.** In the same configuration as for the NOR gate, if LOW state (bit 0) is now associated with  $R_{nl} = +R_0$  and HIGH state (bit 1) to  $R_{nl} = 0$  [convention of Eq. (43)], one obviously gets an OR gate. Of course since the association of HIGH and LOW has been reversed with the previous case, the two settings are incompatible since they correspond to opposite bit assignment.

**AND gate.** We still pin  $F2$  and  $F4$  ( $F2 : \downarrow$  and  $F4 : \downarrow$ ) so that  $F2 = 0$  and  $F4 = 0$  in bit terms. Using Eq. (42) this implies  $F1 F3 = R_{nl}$  which is an AND operation (using the first convention for  $R_{nl}$  bit content). This gate is compatible with the NOR gate but not the OR gate which is produced with a different convention for bit coding.



**NAND gate.** By changing the bit coding of  $R_{nl}$ , the AND gate turns into a NAND gate with the same configuration for  $F2$  and  $F4$ . As mentioned previously, the achievement of a NAND is quit noteworthy since it has functional completeness in Boolean algebra.

**$\overline{AB}$  or  $A \not\Rightarrow B$  ( $A$  not implied by  $B$ ) gate.** We now pin  $F2$  and  $F3$  and choose  $F2 = 1$  and  $F3 = 0$  in bit terms. Then Eq. (42) (first convention) turns into  $\overline{F1} F4 = R_{nl}$ .

**$A + \overline{B}$  or  $A \Leftarrow B$  gate.** Using the same configuration for  $F2$  and  $F3$  as previous gate but exchanging the conventions for the bit content for  $R_{nl}$  leads to an inverted gate: indeed  $\overline{F1} F4 = \overline{R_{nl}}$  implies  $F1 + \overline{F4} = R_{nl}$  by De Morgan theorem.

**$A\overline{B}$  or  $A \not\Leftarrow B$  gate.** We pin  $F2$  and  $F3$  and choose  $F2 = 0$  and  $F3 = 1$  in bit terms. Then Eq. (42) (first convention) turns into  $F1 \overline{F4} = R_{nl}$ .

**$\overline{A} + B$  or  $A \Rightarrow B$  gate.** Choosing the second convention of Eq. (43) with  $F2 = 0$  and  $F3 = 1$  leads to  $F1 \overline{F4} = \overline{R_{nl}}$  which is equivalent to  $\overline{F1} + F4 = R_{nl}$  by De Morgan theorem.

**FALSE gate.** We pin  $F1$  and  $F2$  in parallel configuration. Then  $R_{nl} = 0$  (in terms of resistance not bit) whatever the configuration of  $F3$  and  $F4$ . If we adopt the first convention, the zero resistance translates into bit 0. This is therefore a FALSE gate.

**TRUE gate.** In the same terminals configuration as previous gates, if we adopt the opposite convention for the bit content of  $R_{nl}$ , one then gets a TRUE function.

The six other possible binary operations (out of sixteen) can not be built as easily using a single cross device; this leaves out the NOR and XNOR gates from the list of the basic gates in use in electronic logic (AND, OR, NOR, NAND, XOR, XNOR, buffer, inverter). A way to achieve them would be to chain gates since all Boolean functions can be recovered using only a NOR (or NAND) gate.

We now turn to the two other basic gates, buffer and inverter.

**Buffer.** For that function only one terminal is not pinned. We choose to pin  $F1 = 1$ ,  $F2 = 0$  and  $F4 = 0$ . Using Eq. (42) this implies  $F3 = R_{nl}$ . This can also be realized by pinning  $F1 = 0$ ,  $F2 = 1$  and  $F4 = 1$ . Using Eq. (43) this implies  $\overline{F3} = \overline{R_{nl}}$ .

**Inverter.** When pinning  $F1 = 1$ ,  $F2 = 0$  and  $F4 = 0$ , using Eq. (43) implies  $F3 = \overline{R_{nl}}$ . This can also be realized with Eq. (42) by pinning  $F1 = 0$ ,  $F2 = 1$  and  $F4 = 1$  which leads to  $\overline{F3} = R_{nl}$ .

The NOR, AND,  $A \not\Leftarrow B$ ,  $A \not\Rightarrow B$ , FALSE, buffer and inverter gates use the same bit convention for  $R_{nl}$  (HIGH state for  $R_{nl} = +R_0$ , LOW state for  $R_{nl} = 0$ ) and can be implemented together in the same circuit; since they differ only by the assignment of pinned

terminals, this means that one can easily turn a gate into another one (reprogrammable logic) through local application of an external magnetic field or through spin transfer torque.

In summary, the non-local cross with four ferromagnetic terminals is a versatile spintronics device which can be used either as a standard spin valve: when some symmetry requirements are met, it displays the important property of an infinite GMR for the non-local resistance; it can be immunized against offset voltages observed in many non-local setups. Finally, it can be used as a magneto-logic gate as well as probe the Direct Spin Hall Effect.

### III. VAN DER PAUW CROSS WITH TWO OR THREE FERROMAGNETS.

Which principles should guide us in order to achieve large signals in non-local setups? As stressed time and again in the literature<sup>15,17,23</sup>, they are quite simple: larger spin accumulations in the paramagnet can be generated if (i) the paramagnet volume in which spins can relax is small (in comparison with  $l_N^3$ ) and if (ii) spin back-flow to the ferromagnets is hindered by large enough interface resistances.

One can classify geometries as open or closed according to the (non-)fulfillment of these prescriptions<sup>23</sup>: it has been argued in the latter reference that at small enough volume and for large enough tunnel barriers the spin accumulation can be greatly enhanced because it then scales with the (large) interface resistances  $R_c$ . But whenever the geometry is open, the spin accumulation scales with the much smaller spin resistance of the paramagnetic channel  $R_N$  which in practice is often in the Ohm range. Although prescriptions (i) and (ii) have been stressed repeatedly, spin leakage in the current or voltage probes is often overlooked although they may alter significantly the signal (an example is provided in Appendix B).

The goal of this section is to examine the transition from open to closed geometries as a strategy to enhance the non-local signal used for instance to read or sense 1 bit, and to study its interaction with the doubling of spin injector electrodes. The signal generated in our cross geometry will be systematically compared with the spin voltage in the standard lateral device.

The following general features will be a guide: a large signal requires a large total effective spin resistance for the device  $R_{eff}$ . Since the latter can be interpreted as resulting from the addition of four spin resistances  $R_{eff,i}$  ( $i = 1 - 4$ ) in parallel corresponding to the four arms of the cross, one will expect a small signal whenever one or several of these spin resistances

are significantly smaller than the others since they will short the other arms. If all effective spin resistances  $R_{eff,i}$  are commensurate, in order to have a large  $R_{eff,i}$ , it is then necessary that  $R_{c,i} \gg R_N$  simultaneously for all four terminals as explained in section II B 2.

(As an aside remark we wish to bring to the reader's attention that the expressions derived in the literature<sup>47,48</sup> for the bipolar spin switch transistor of Johnson remarkably show the enhancements characteristic of closed geometries over open geometries with a typical scaling with interface resistance  $R_c$  at small enough distance between the ferromagnetic terminals. As discussed in Appendix B this is due to neglecting spin leakage to the current drain; when this leakage is taken into account, the signal is actually found to scale with  $R_N$  as in open geometries.)

### A. Van der Pauw cross with two ferromagnets.

#### 1. Open geometry with two ferromagnets.

For the sake of comparison we first consider the case when the detector electrodes are paramagnets as in the cross arms  $I - IV$ . When  $F1$  and  $F3$  are identical ferromagnets at the same distance of origin ( $l_1 = l_3$ ) while  $F2$  and  $F4$  are identical paramagnets with spin resistance  $R_N$ , the spin resistance mismatches at each terminal are

$$X_1 = X_3 = X; X_2 = X_4 = 1 \quad (46)$$

while:

$$\widetilde{PR}_1 = \widetilde{PR}_3 = \widetilde{PR} \quad (47)$$

$$\widetilde{PR}_2 = \widetilde{PR}_4 = 0 \quad (48)$$

and defining  $\delta_F^\pm$  for arms  $I$  and  $III$ :

$$\delta_F^\pm = \delta_{1/3}^\pm = \frac{(X+1)}{2} \exp l_1 \pm \frac{(X-1)}{2} \exp -l_1. \quad (49)$$

$$\delta_{2/4}^\pm = \exp l_{2/4} \quad (50)$$

so that:

$$\begin{aligned}
R_{nl} &= \frac{\sigma_1 \sigma_3 R_N \left( \widetilde{PR} \right)^2}{2 \left[ \delta_F^- \delta_F^+ + \left( \delta_F^+ \right)^2 \right]} \\
&= \frac{\sigma_1 \sigma_3 R_N \left( \widetilde{PR} \right)^2}{\left[ (1+X)^2 \exp l + (X^2 - 1) \right]}
\end{aligned} \tag{51}$$

where  $l = 2L_1/l_N$  the total length separating the ferromagnetic terminals (in units of the spin diffusion length in the paramagnet) and where we have defined  $\sigma_1 = \pm 1$ ,  $\sigma_3 = \pm 1$  to index the majority spin directions of electrodes  $F1$  and  $F3$  (relative to an absolute axis).

In the standard geometry, since one has a spin valve it is customary to quote the resistance variation when one ferromagnet magnetization is switched; this is twice the maximum value  $R_P$ :

$$\delta R_{nl,0} = R_P - R_{AP} = \frac{2 R_N \left( \widetilde{PR} \right)^2}{\left[ (1+X)^2 \exp l + (X^2 - 1) \right]}. \tag{52}$$

This generalizes the expression found in the literature for the non-local resistance variation: more precisely the result quoted by Jedema and coll.<sup>4</sup> corresponds to the case of vanishing interface resistance, so that  $X = R_F/R_N$  ( $= 1/M$  using Jedema and coll. notations<sup>4</sup>).

Although the geometry is open (in the sense that spin current can leak easily since terminals  $F2/F4$  do not hinder its flow ( $X_2 = X_4 = 1$ )), it is interesting to observe that the larger the resistance mismatch  $X$  at the ferromagnets, the larger the signal (for instance, if we set  $P = 1$  in Eq. (52) one gets that  $\delta R_{nl,0} \propto X^2 / \left[ (1+X)^2 \exp l + (X^2 - 1) \right]$  which is an increasing function of  $X$ ). This means that large resistance mismatches are already beneficial and increase the spin accumulation although there is some spin leakage.

The largest value is at short distance when the denominator of Eq. (52) is  $2X(X+1)$ . In the tunneling regime  $X \gg 1$ ,  $\delta R_{nl,0} \propto R_N$ . In the opposite limit  $X \ll 1$ , the signal will be even smaller since it scales as  $R_c + R_F \ll R_N$ :

$$\delta R_{nl,0}(l \rightarrow 0) \geq \inf(P_c, P_F)^2 (R_c + R_F)$$

and

$$\delta R_{nl,0}(l \rightarrow 0) \leq \sup(P_c, P_F)^2 (R_c + R_F).$$

Such a scaling which is at most  $\sim R_N$  or even below ( $\ll R_N$ ) is characteristic of open geometries.

Let us compare to the non-local resistance variation for the lateral geometry with identical parameters (same distance  $l$  between injector and detector, same spin resistance mismatch  $X$  at ferromagnets, same cross-section for the paramagnet channel connecting the ferromagnets)<sup>6</sup>:

$$\delta R_{nl,lateral} = \frac{4R_n \widetilde{PR}^2}{[(2X+1)^2 \exp l - \exp -l]}. \quad (53)$$

We plot on Fig. 6 the ratio of the cross signal versus the one in the lateral geometry:

$$m(X, l) = \frac{\delta R_{nl,0}}{\delta R_{nl,lateral}} = \frac{[(2X+1)^2 \exp l - \exp -l]}{2[(1+X)^2 \exp l + (X^2 - 1)]}. \quad (54)$$

In the short distance limit:

$$m(X, l \rightarrow 0) \rightarrow 1$$

which is expected since the cross and the lateral geometries are then identical. At large distance, the signal is larger in the cross geometry whenever  $X > 1/\sqrt{2}$  (which includes the tunneling regime at the ferromagnets and also part of the transparent regime). The impact of a large value of  $X$  is moderate (about 10 % between  $X = 10 - 100$ ) in stark contrast to what we will observe in a closed geometry. The general behaviour is easy to understand: in the tunneling regime spin current leakage is less pronounced in the cross geometry since the paramagnetic drain is further away (while the detector ferromagnet and paramagnetic counter-electrodes are at the same distance). In the transparent regime, by a similar reasoning one gets the opposite.

## 2. Spin confining geometry with two ferromagnets.

It has been argued by Jaffres and coll.<sup>23</sup> that spin confinement tends to increase non-local signals (resistance or voltage) since spin accumulation is stronger whenever spin leaking is hindered by large tunnel barriers. We can study this by considering that electrodes  $F2$  and  $F4$  are normal paramagnets but that there is a resistance mismatch

$$Y = \frac{R_{N,0} + R_{c,0}}{R_N} \quad (55)$$

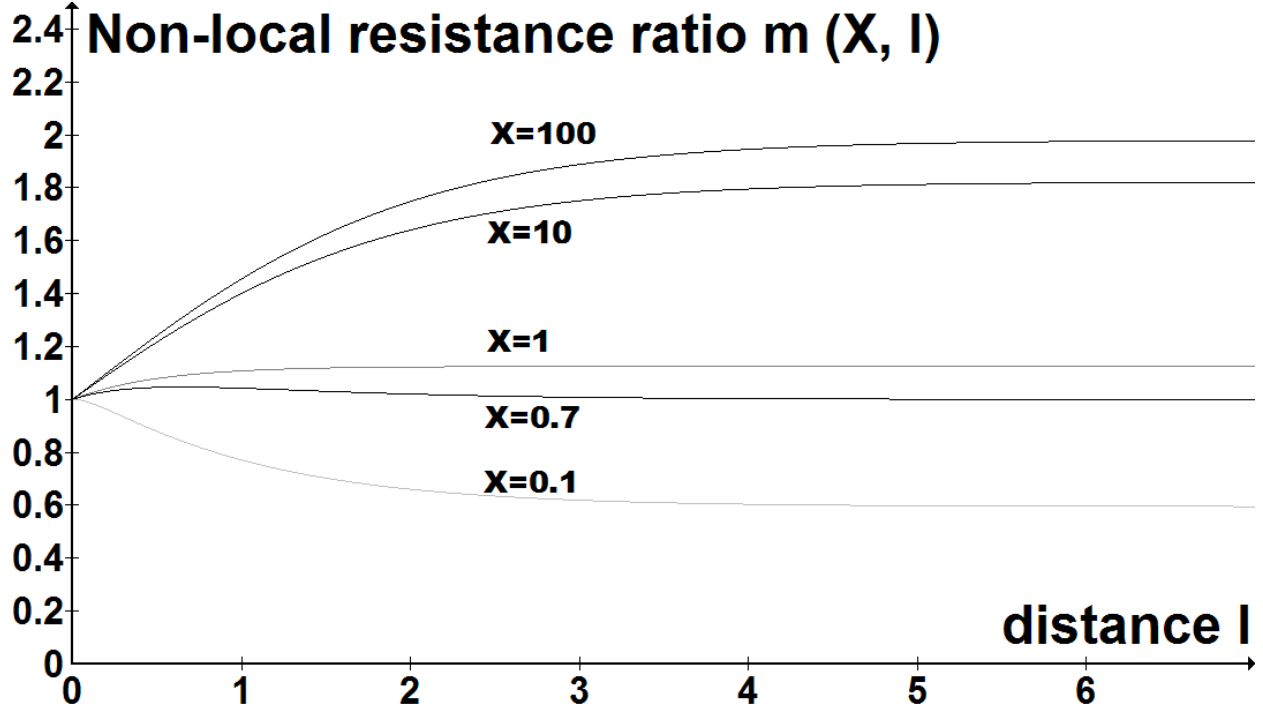


Figure 6. Non-local resistance in cross setup compared with lateral setup.  $l$  is the distance in units of spin diffusion length between injector and detector in either setup. In tunneling regime ( $X > 1$ ) and in part of the transparent regime, the signal is larger in the cross geometry.

where  $R_{N,0}$  and  $R_{c,0}$  are the spin resistance of  $F2$  and  $F4$  and the interface resistance between them and the central cross. One can therefore interpolate between an *open geometry* (the standard geometry, i.e.  $Y = 1$ ) and a *closed geometry* ( $Y \gg 1$  and  $X \gg 1$ ). We will now study the transition from one regime to the other.

Let us assume that terminals  $F1$  and  $F3$  are identical ferromagnets at identical distance  $l_1 = l_3 = l_0$  of the cross center  $O$ , that  $F2$  and  $F4$  are identical paramagnets also at the same distance  $l_2 = l_4 = l_0$  from  $O$  ( $ABCD$  is then a square); the total distance from the injector  $F1$  to the detector  $F3$  is  $l = 2l_0$ .

We also set:

$$X_1 = X_3 = X; X_2 = X_4 = Y \quad (56)$$

and:

$$\widetilde{PR}_1 = \widetilde{PR}_3 = \widetilde{PR}, \quad (57)$$

$$\widetilde{PR}_2 = \widetilde{PR}_4 = 0. \quad (58)$$

There are several obvious ways to create this spin resistance mismatch  $Y$ : one is to deposit a tunnel barrier between the terminals  $F2$  and  $F4$  and the central cross ( $R_{c,0} \neq 0$ ); another is to use the same paramagnet for both  $F2$ ,  $F4$  and the central cross but have a different cross-section ( $R_{c,0} = 0$  but  $R_{N,0} = \rho_N^* l_N / A_{N,0} \neq R_N = \rho_N^* l_N / A_N$ ).

One ends up with:

$$R_{nl} = \frac{\sigma_1 \sigma_3 R_N \left( \widetilde{PR} \right)^2}{2 \left[ \delta_F^- \delta_F^+ + \frac{\delta_N^-}{\delta_N^+} (\delta_F^+)^2 \right]} \quad (59)$$

where

$$\delta_F^\pm = \delta_{1/3}^\pm, \quad \delta_N^\pm = \delta_{2/4}^\pm.$$

[The definitions for  $\delta_i^\pm$  are given in Eq. (6).]

Let us define

$$\begin{aligned} \Delta_{2F}(X, Y, l) = & 2 [(Y+1) \exp l_0 + (Y-1) \exp -l_0]^{-1} \\ & \{ (Y+1) \exp l_0 [(X+1)^2 \exp 2l_0 + (X^2 - 1)] \\ & - (Y-1) \exp -l_0 [(X-1)^2 \exp -2l_0 + (X^2 - 1)] \} \end{aligned} \quad (60)$$

and

$$\Delta_0(X, l) = [(2X+1)^2 \exp 2l_0 - \exp(-l)] \quad (61)$$

so that:

$$\delta R_{nl} = \frac{4 R_N \left( \widetilde{PR} \right)^2}{\Delta_{2F}} \quad (62)$$

The signal will be largest at short distance for which:

$$\inf (P_c, P_F)^2 R_N \frac{XY}{X+Y} \leq \delta R_{nl}(l \rightarrow 0)$$

and:

$$\delta R_{nl}(l \rightarrow 0) \leq \sup (P_c, P_F)^2 R_N \frac{XY}{X+Y}$$

so that  $\delta R_{nl}(l \rightarrow 0)$  scales as  $R_N \frac{XY}{X+Y}$ .

Therefore in the strong tunneling regime ( $X \gg 1$  and  $Y \gg 1$ ),  $\delta R_{nl}$  scales as

$$\inf(X, Y) R_N \gg R_N$$

yielding much larger signals than in the lateral geometry. But whenever either of  $X$  or  $Y$  is in the transparent regime,  $\delta R_{nl}$  will scale as the smaller of the two and so will be at most at

the scale of  $R_N$  as predicted for open geometries (see IIB 2). In order to check the impact of increasing the spin resistance mismatches at the various terminals to the signal in the lateral geometry we consider a non-local resistance ratio  $m(X, Y, l)$  per:

$$m(X, Y, l) = \frac{\delta R_{nl}(X, Y, l)}{\delta R_{nl,lateral}(X, l)} \quad (63a)$$

$$= \frac{\Delta_0(X, l)}{\Delta_{2F}(X, Y, l)}. \quad (63b)$$

The relative increase of the signal is then  $m(X, Y, l) - 1$ . At  $l = 0$ :

$$m(X, Y, l = 0) \longrightarrow \frac{Y (X + 1)}{X + Y}. \quad (64)$$

At large distance:

$$m(X, Y, l) \longrightarrow \frac{(2X + 1)^2}{2(X + 1)^2} \quad (65)$$

which is larger than 1 whenever  $x \geq 1/\sqrt{2}$  and more generally belongs to the interval  $[0.5; 2]$ .

The large distance behaviour is of course less interesting since the non-local signal is weaker, so we will discuss in detail only the short distance behaviour.

At short distances, large values are achieved whenever both parameters  $X$  and  $Y$  are large (spin confining regime, closed geometry). For  $X \gg Y \gg 1$ ,  $m(X, Y, l = 0) \sim Y$ ; while for  $Y \gg X \gg 1$ ,  $m(X, Y, l = 0) \sim X+1$ ; finally for  $X \sim Y \gg 1$ ,  $m(X, Y, l = 0) \sim (X + 1) / 2$ . This third situation is probably the easiest to achieve: indeed if we want to gain at least an order of magnitude the other cases would imply that one of the mismatches is at least two order of magnitudes larger while when  $X \sim Y \gg 1$  they both have the same magnitude. For a realistic value  $X \sim 10$  in the weak tunneling regime (where the contact resistance is usually metallic) this yields an enhancement by a factor up to 5 (see Section IV for a discussion of realistic parameters  $X \sim 10$ ).

We plot in the next figures (Fig. 7-9) the length dependence for various values of  $X$  and  $Y$  in the weak tunneling regime (keeping moderate values  $(X, Y) \leq 10$ ) covering the three situations described in the previous paragraph (larger  $X$ , larger  $Y$  or equal  $X = Y$ ).



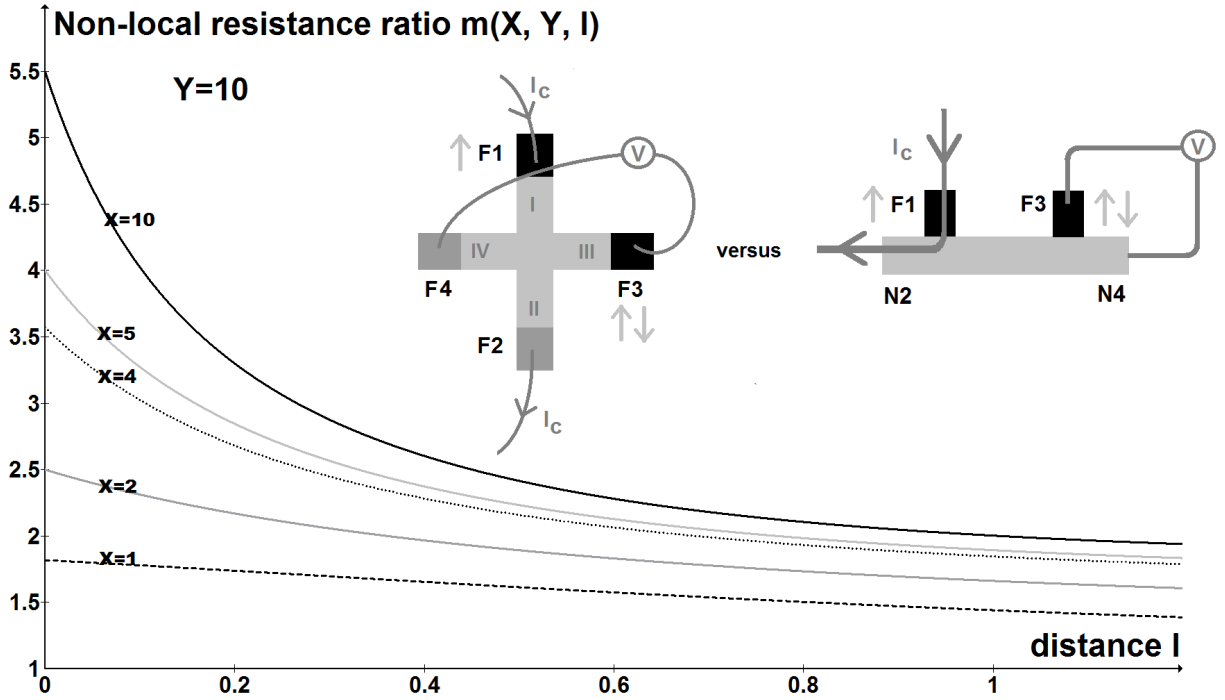


Figure 7. Ratio of non-local resistance variation in cross versus lateral setup as a function of distance  $l$  between injector  $F1$  and detector  $F3$  (in units of spin diffusion length  $l_N$ ) for spin resistance mismatches  $Y = 10$  (at terminals  $F2/F4$ ) and  $X = 1 - 10$  (at terminals  $F1/F3$ ).  $F1$  and  $F3$  are identical ferromagnetic metals while  $F2$  and  $F4$  are paramagnets in this section with a spin resistance mismatch with the central paramagnet.

Still in the weak tunneling regime we also plot on Fig. 10 the non-local resistance ratio for fixed distance  $l = 0.1 - 0.3$  as a function of spin resistance mismatch  $X$  (assuming  $Y = X$ ). Values in the range  $l = 0.1 - 0.3$  are quite reasonable experimentally (for instance for  $Cu$  at ambient temperature  $l_N = 300 \text{ nm}$  while a distance  $L = 100 \text{ nm}$  is within reach of lithography). At such distances the enhancement can still be several hundred of percents as can be seen in Fig. 10 unless  $X < 1$  (transparent regime).

The transparent regime is less interesting in terms of an increase of the signal since the geometry is now open. Yet whenever  $Y$  is larger than 1, even if  $X$  is in the transparent regime, one still gets an increase of the signal at short distance (Fig. 11). But if both parameters are smaller than unity, the signal gets reduced by several tens of percent when

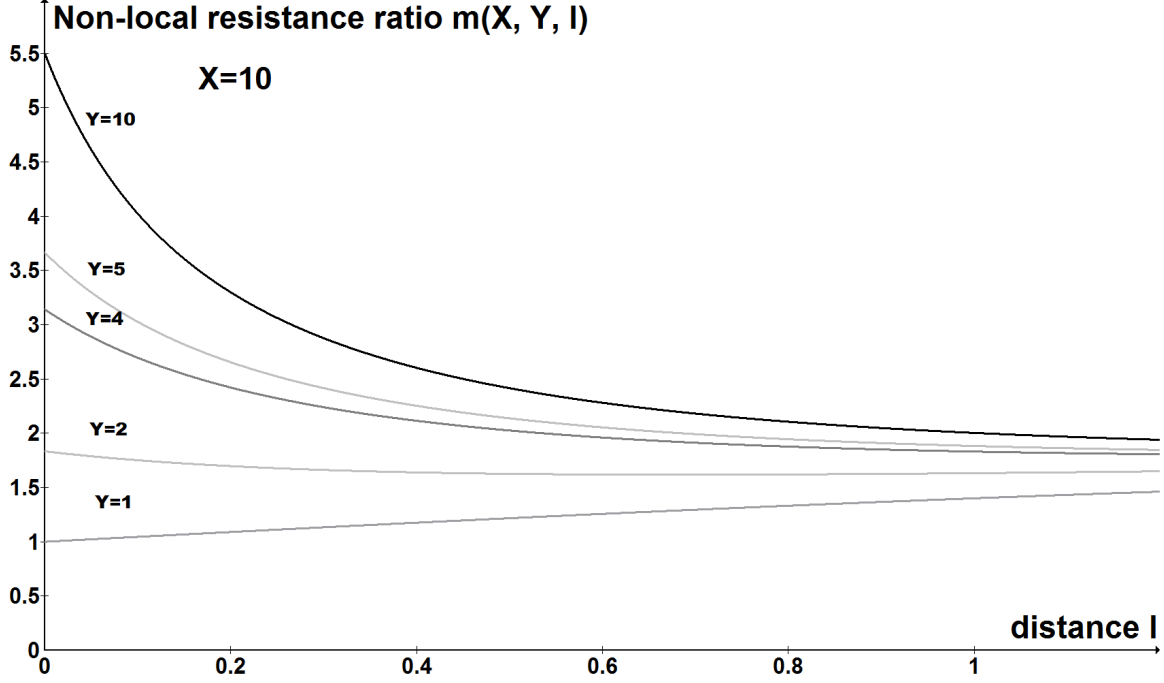


Figure 8. Ratio of non-local resistance variation in cross versus lateral setup as a function of distance  $l$  between injector  $F1$  and detector  $F3$  (in units of spin diffusion length  $l_N$ ) for spin resistance mismatches  $Y = 1 - 10$  (at terminals  $F2/F4$ ) and  $X = 10$  (at terminals  $F1/F3$ ).  $F1$  and  $F3$  are identical ferromagnetic metals while  $F2$  and  $F4$  are paramagnets in this section with a spin resistance mismatch with the central paramagnet.

compared against the lateral setup as can be seen on Fig. 12-13.

To summarize this section devoted to the cross geometry with two ferromagnets (one as charge source and spin injector, the other as spin accumulation detector), we have confirmed the impact on spin confinement of tunnel barriers even of moderate strength ( $X = 10$  might correspond to  $R_c \sim 10 \Omega$  since  $R_N$  is usually in the Ohm range). In the transparent limit, the device under-performs when compared to the standard lateral spin valve and should be avoided.

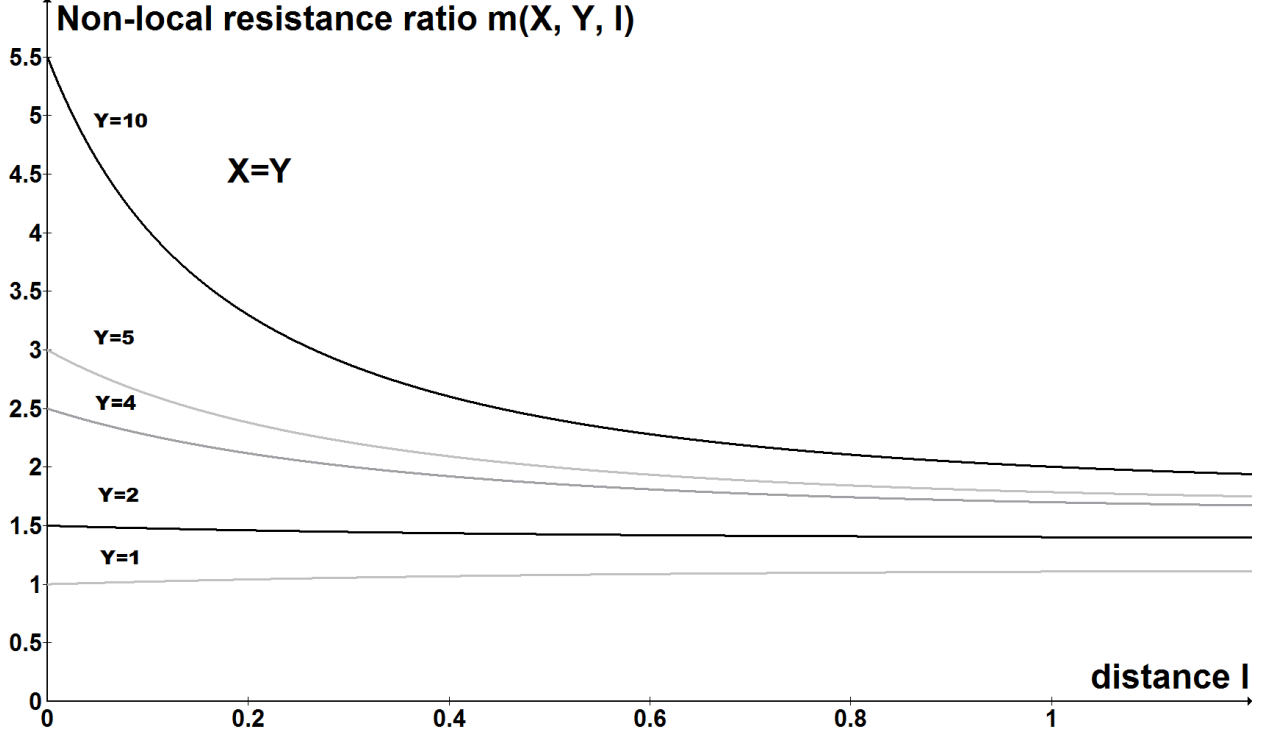


Figure 9. Ratio of non-local resistance variation in cross versus lateral setup as a function of distance  $l$  between injector  $F1$  and detector  $F3$  (in units of spin diffusion length  $l_N$ ) for spin resistance mismatches  $Y = 1 - 10$  (at terminals  $F2/F4$ ) and  $X = Y$  (at terminals  $F1/F3$ ).  $F1$  and  $F3$  are identical ferromagnetic metals while  $F2$  and  $F4$  are paramagnets in this section..

### B. Van der Pauw cross with three ferromagnets.

We now use two ferromagnets as spin injectors. When the (charge) source and drain are in antiparallel orientation the signal is enhanced because both ferromagnets acts as spin sources. But when they are parallel, one is a spin source and the other a spin sink so that spin accumulation is reduced. If the electrodes are identical ferromagnets, then the signal will be doubled when compared with the case where there is only one spin injector electrode. This is exactly what we found since  $R_{nl} \propto P_{eff,1} - P_{eff,2} = 2P_{eff,1}$  if electrodes are identical and antiparallel.

We assume that  $F1 - F2 - F3$  are identical electrodes ( $F1$  and  $F2$  antiparallel while  $F3$  can switch from one orientation to the other) and  $F4$  is a paramagnet. The spin resistance mismatches are set as:

$$X_1 = X_2 = X_3 = X; X_4 = Y. \quad (66)$$

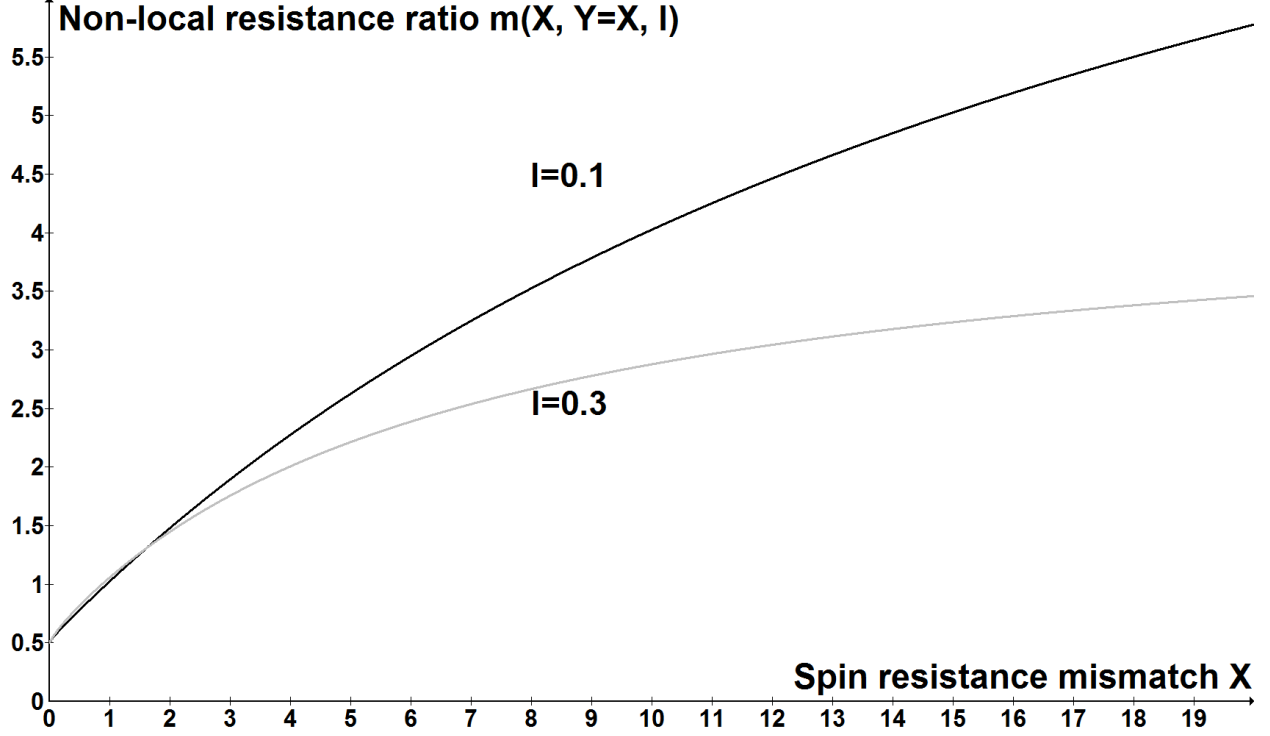


Figure 10. Non-local resistance ratio in cross versus lateral setup at fixed distances  $l = 0.1 - 0.3$  between injector  $F1$  and detector  $F3$  (in units of spin diffusion length  $l_N$ ) for spin resistance mismatches  $X = Y$ .

We also assume  $l_1 = l_2 = l_3 = l_4$  and use the same definitions as in previous section:

$$\widetilde{PR}_1 = \widetilde{PR}_2 = \widetilde{PR}_3 = \widetilde{PR} \quad (67)$$

$$\widetilde{PR}_4 = 0 \quad (68)$$

Then:

$$R_{nl} = \frac{2\sigma_1\sigma_3 R_N (\widetilde{PR})^2}{\left[3\delta_F^-\delta_F^+ + \frac{\delta_N^-}{\delta_N^+} (\delta_F^+)^2\right]} \quad (69)$$

where  $\sigma_1 = \pm 1$  and  $\sigma_3 = \pm$  refer to the majority spin direction of ferromagnets  $F1$  and  $F3$  relative to an absolute axis.

Let us define

$$\begin{aligned} \Delta_{3F}(X, Y, l) = & [2(Y+1) \exp l_0 + 2(Y-1) \exp -l_0]^{-1} \\ & \times \{ (Y+1) \exp l_0 [2(X+1)^2 \exp 2l_0 - (X-1)^2 \exp -2l_0 + (X^2-1)] \\ & - (Y-1) \exp -l_0 [2(X-1)^2 \exp -2l_0 - (X+1)^2 \exp 2l_0 + (X^2-1)] \} \end{aligned} \quad (70)$$

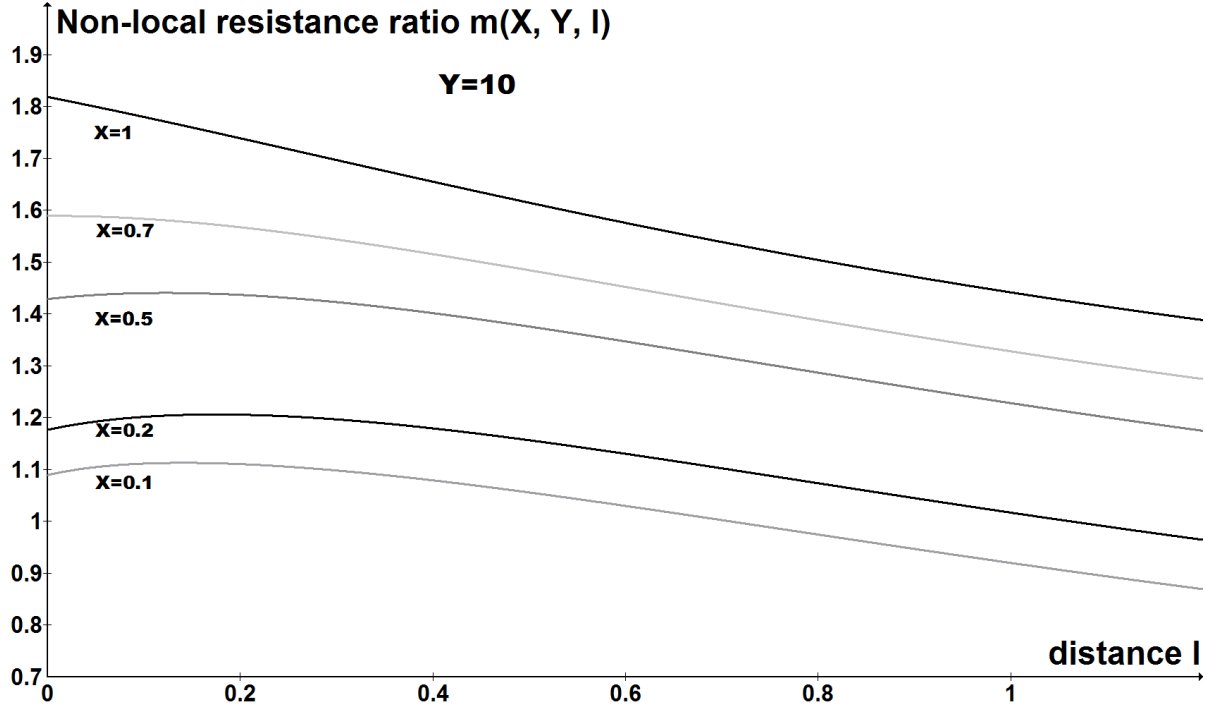


Figure 11. Ratio of non-local resistance variation in cross versus lateral setup as a function of distance  $l$  between injector  $F1$  and detector  $F3$  for spin resistance mismatches  $Y = 10$  (at terminals  $F2/F4$ ) in the weak tunneling regime and  $X = 0.1 - 1$  (at terminals  $F1/F3$ ) in the transparent regime.

Then:

$$\delta R_{nl} = R_P - R_{AP} = \frac{4 R_N \left( \widetilde{PR} \right)^2}{\Delta_{3F}} \quad (71)$$

where  $P/AP$  refer to the direction of  $F3$  relative to  $F1$ . Therefore the non-local resistance roughly scales as:

$$\delta R_{nl} \sim \frac{C R_N X^2}{\Delta_{3F}(X, Y, l)} \quad (72)$$

(where  $C$  is a constant; we have used upper and lower bounds on  $\left( \widetilde{PR} \right)^2$  as in section III A above).

The largest signal is found at small distance when:

$$\delta R_{nl} \longrightarrow R_N \frac{XY}{3Y + X}.$$

We observe a characteristic quadratic dependence of the numerator against a linear one in the denominator as a function of spin resistance mismatch: this is what ensures that in the

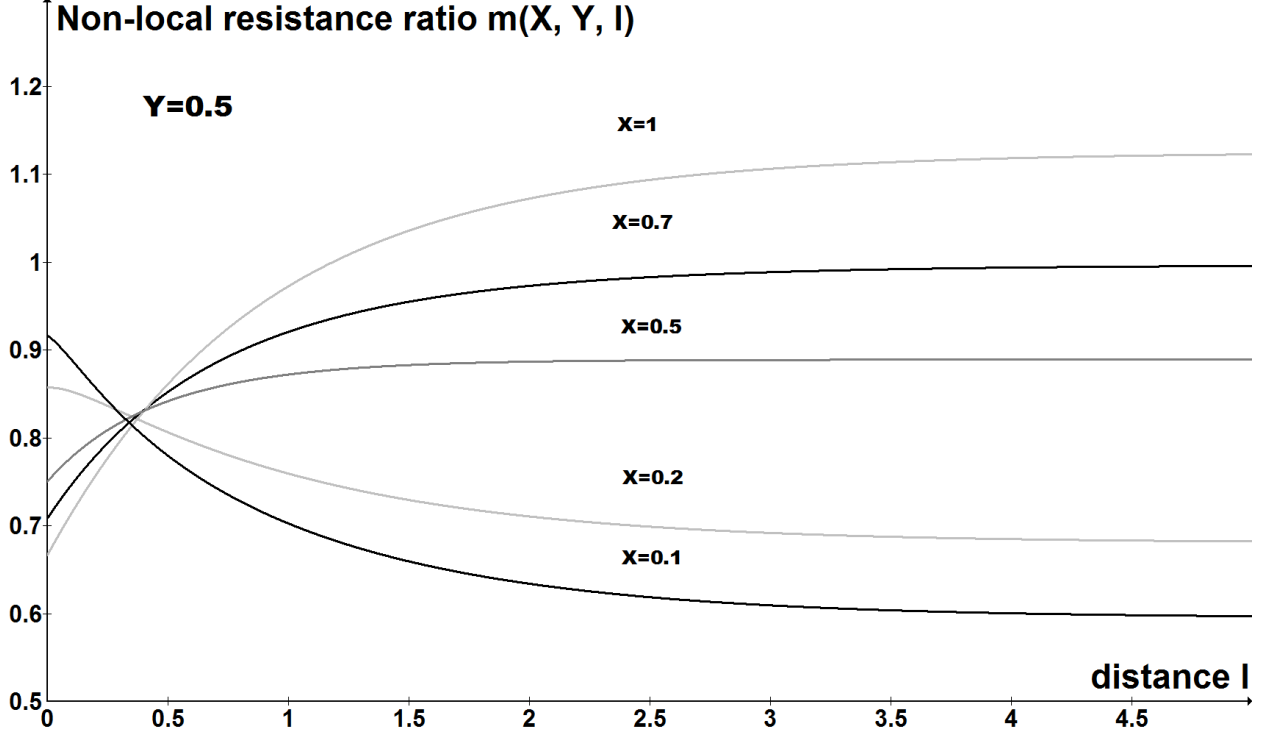


Figure 12. Ratio of non-local resistance variation in cross versus lateral setup as a function of distance  $l$  between injector  $F1$  and detector  $F3$  for spin resistance mismatches  $Y = 0.5$  (at terminals  $F2/F4$ ) and  $X = 0.1 - 1$  (at terminals  $F1/F3$ ). All electrodes are in the transparent regime.

tunneling limit,  $\delta R_{nl}$  can scale as  $R_c$  (here as the smaller of  $R_N X \gg R_N$  or  $R_N Y \gg R_N$ ). But as soon the geometry is open due to one or both spin resistance mismatches in the transparent limit,  $\delta R_{nl}$  scales as  $R_n$  or below.

The ratio of the non-local resistance with regards to the standard lateral setup is then:

$$m(X, Y, l) = \frac{\Delta_0}{\Delta_{3F}} \quad (73)$$

By varying the spin resistance mismatch  $Y$  at terminal  $F4$  one can interpolate between an open geometry ( $Y \leq 1$ ) and a closed one ( $Y \gg 1$  while  $X \gg 1$ ).

Let us make some general comments comparing the three ferromagnets cross setup with the two ferromagnets cross discussed in the previous section III A 2.

When  $X = Y$ , one gets  $\Delta_2 = 2\Delta_3$  so that

$$m_{3F}(X, Y = X, l) = 2m_{2F}(X, Y = X, l).$$

The non-local signal is exactly twice that found for the setup with two ferromagnets. Indeed when all terminals have the same spin resistance mismatch, the effective resistances  $R_{eff}$  are

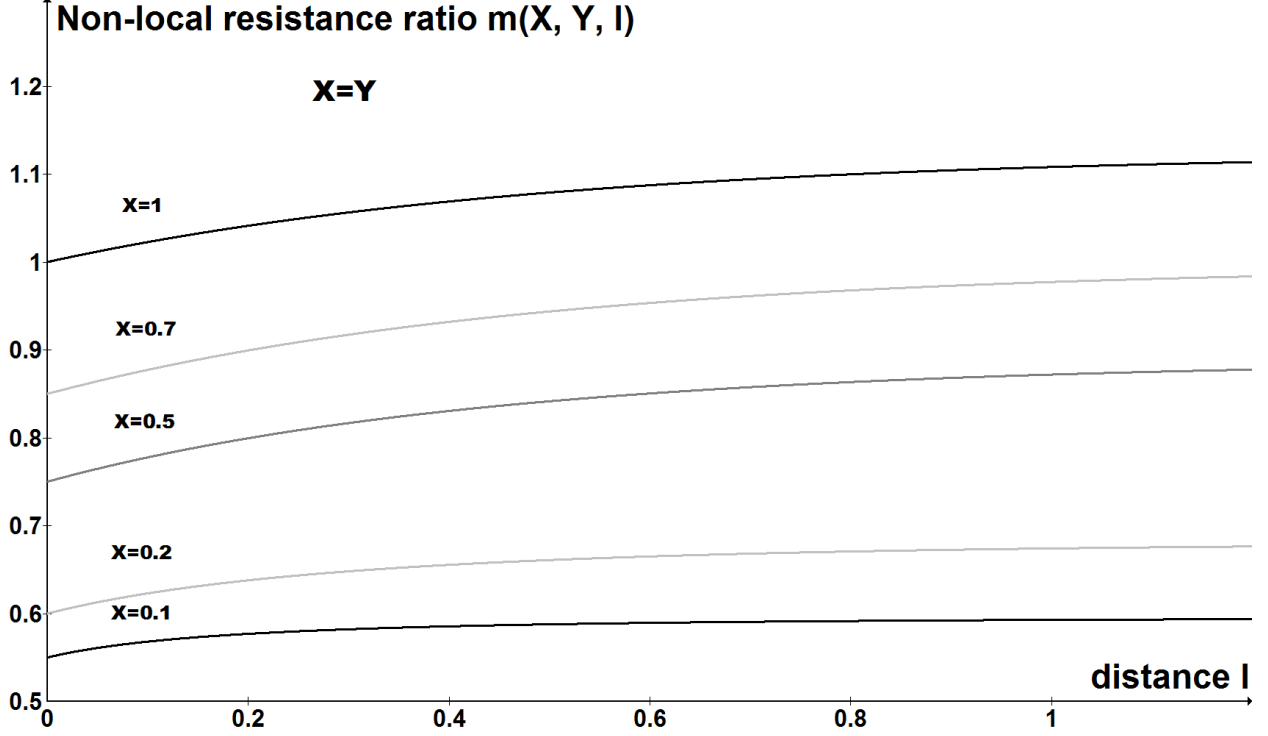


Figure 13. Ratio of non-local resistance variation in cross versus lateral setup as a function of distance  $l$  between injector  $F1$  and detector  $F3$  for spin resistance mismatches  $Y = 0.1 - 1$  (at terminals  $F2/F4$ ) and  $X = Y$  (at terminals  $F1/F3$ ). All electrodes are in the transparent regime.

identical in both setups since the parameters are identical; but the total effective polarization at injector  $P_{eff,1} - P_{eff,2} = 2P_{eff,1}$  is doubled in the 3 ferromagnet setup ( $P_{eff,2} = 0$  in the two ferromagnet setup).

For  $X > Y$  the effective resistance (see II B 1) for the 3 ferromagnet setup becomes larger than that of the 2 ferromagnet setup (see III A 2; this is quite clear since  $R_{eff,i}(X_i)$  is an increasing function of  $X_i$  so that

$$R_{eff,3F} = \left[ \frac{3}{R_{eff,1}(X)} + \frac{1}{R_{eff,1}(Y)} \right]^{-1} >$$

$$R_{eff,2F} = \left[ \frac{2}{R_{eff,1}(X)} + \frac{2}{R_{eff,1}(Y)} \right]^{-1}.$$

This means that the non-local signal will be more than doubled with respect to the 2 ferromagnet setup studied in III A 2. This effect does not require a totally closed geometry to occur.

For  $X < Y$ , the 3 ferromagnet geometry is by the same arguments less spin confining since we have three terminals with a smaller spin resistance against only two in the 2 ferro-

magnet setup. However the doubling of effective polarizations remain which may mitigate the decrease of effective spin resistance. But as a rule in order to achieve stronger signals one should seek the condition  $X \geq Y$ .

1. *Open geometry* ( $Y = 1$ ).

We first consider the case when the paramagnetic terminal has no tunnel barrier but is perfectly matched in terms of spin resistance to the central cross. This is in the terminology of Ref.<sup>23</sup> an open geometry. Nevertheless as in the 2 ferromagnet geometry studied in III A, we will find that spin accumulation can still be enhanced even though not all the terminals are tunnel barriers.

The spin resistance mismatch  $Y$  is set to unity:  $X_4 = Y = 1$ .

The non-local resistance ratio to the two ferromagnet cross (with  $Y = 1$ ) is then:

$$m(X, Y = 1, l) = 2 \left[ (2X + 1)^2 \exp l - \exp -l \right] \quad (74)$$

$$\times \left[ 2(X + 1)^2 \exp l - (X - 1)^2 \exp -l + (X^2 - 1) \right]^{-1} \quad (75)$$

At large distance its limit is:

$$m(X, Y = 1, l) \longrightarrow \left( \frac{2X + 1}{X + 1} \right)^2. \quad (76)$$

which is always larger than 1 whether  $X$  is in the spin confining ( $X > 1$ ) or the transparent regime ( $X < 1$ ).

At small distance the ratio tends to:

$$m(X, Y = 1, l) \longrightarrow 4 \frac{X + 1}{X + 3} \quad (77)$$

which is larger than 2 and can be as large as 4 in the tunneling regime (provided  $X > 1$ ) and is comprised in the interval  $[4/3; 2]$  for  $X \leq 1$  (transparent regime).

So although the geometry is open, the use of tunneling junctions does have some spin confining effect, which here is reinforced by the use of two injector electrodes resulting in an enhancement of the non-local resistance which can be up to 300% increase in the tunneling regime. For a distance  $l = 0.2$  and  $X = 2 - 10$ ,  $m = 2.45 - 3.30$  (145 - 230 % increase). As can be seen in the next figure (Fig. 14) the enhancement can be seen at all distances so that this geometry is already better than the lateral one in terms of spin confinement.



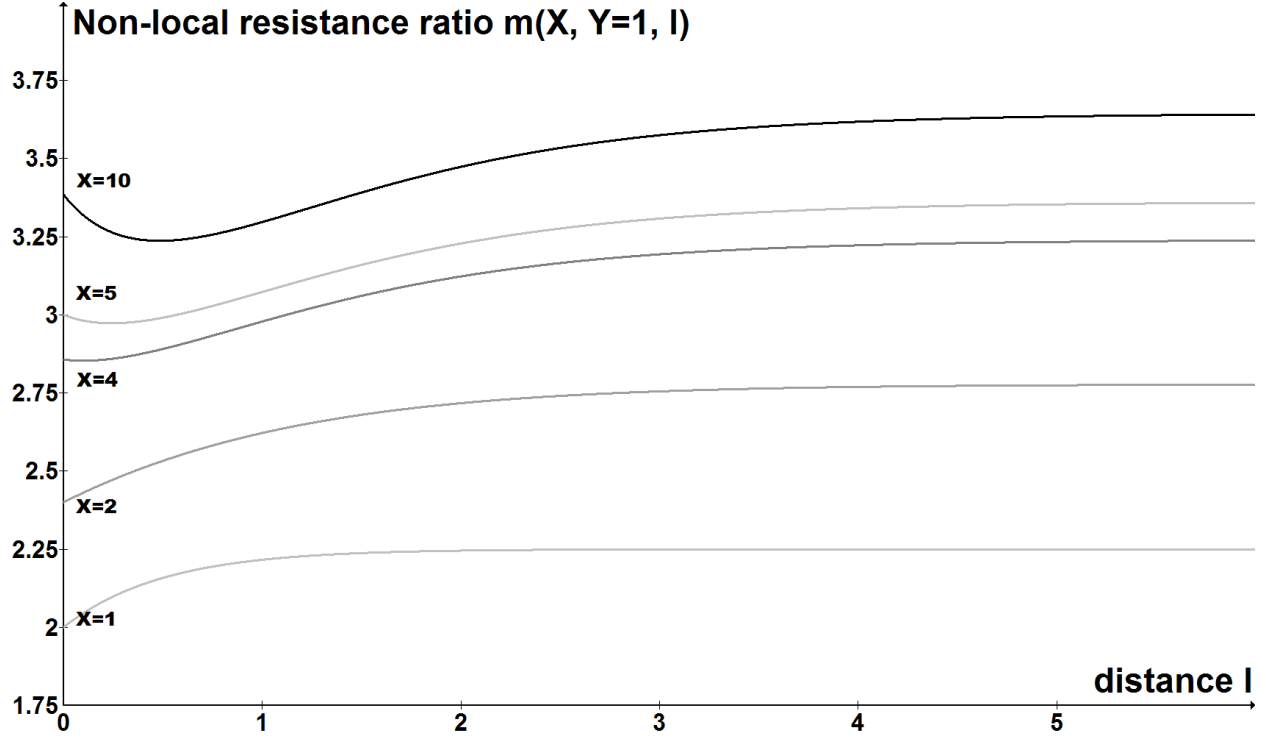


Figure 14. Ratio of non-local resistance in cross geometry (with three ferromagnets) versus lateral setup as a function of distance  $l$  between injector and detector. Spin resistance mismatch is  $X = 1 - 10$  at terminals  $F1 - F2 - F3$  (weak tunneling regime).

In the transparent regime ( $X < 1$ ), the doubling effect coming from the two spin injectors compensate partly the spin leakage due to transparent junctions so that there is always at least a 33 % increase at short distance. The enhancement is still present at large distance and can then reach up to 125 % as can be seen in Fig.15.

## 2. General case.

We now conjugate the effects of spin confinement by tunnel barriers and the doubling of injector terminals. At small distance one gets:

$$m(X, Y = 1, l) \longrightarrow 4Y \frac{X + 1}{X + 3Y} \quad (78)$$

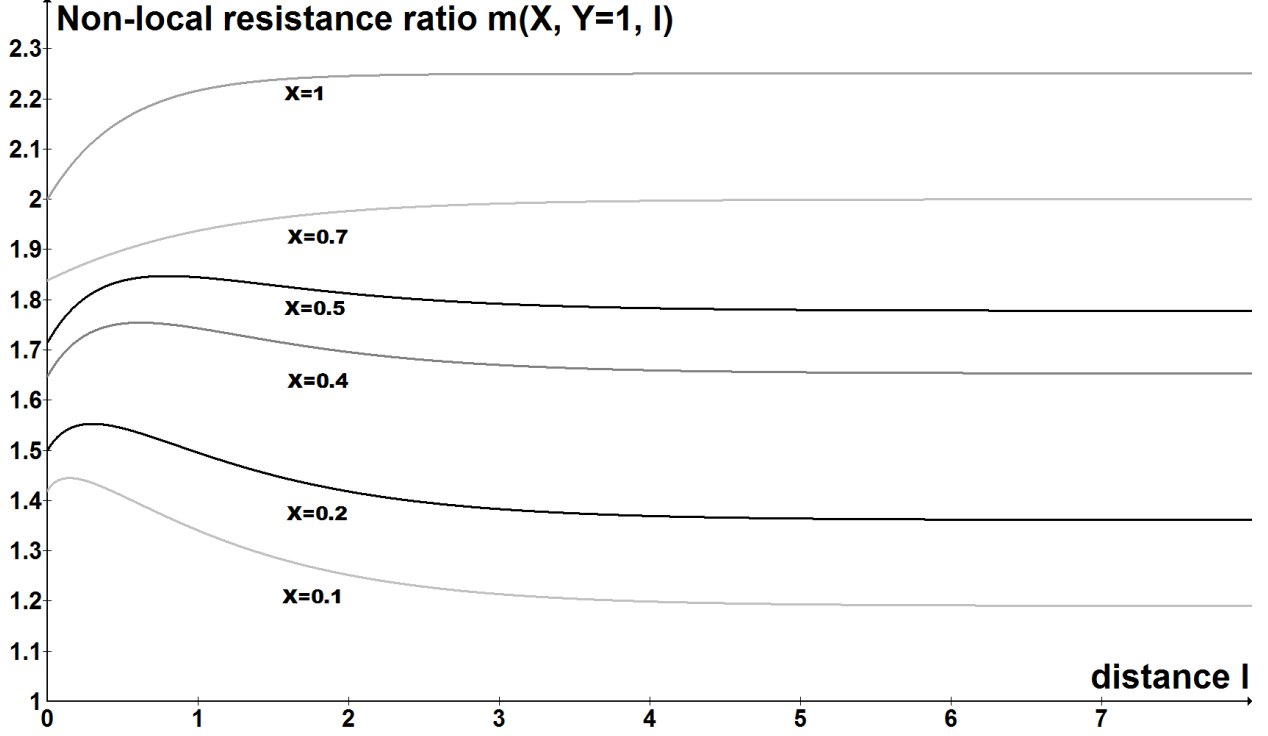


Figure 15. Ratio of non-local resistance in cross geometry (with three ferromagnets) versus lateral setup as a function of distance  $l$  between injector and detector. Spin resistance mismatch is  $X = 0.1 - 1$  at terminals  $F1 - F2 - F3$  (transparent regime). There is still an enhancement of the signal.

while at large distances:

$$m(X, Y = 1, l \rightarrow \infty) \rightarrow \left( \frac{2X + 1}{X + 1} \right)^2 < 4. \quad (79)$$

Large values are achieved whenever both parameters  $X$  and  $Y$  are large (both in the tunneling regime).

Let us focus first on that tunneling regime.

There are then three interesting limits:

(i)  $X \gg Y \gg 1$ , which implies  $m(X, Y, l = 0) \sim 4Y$ ; (note also that the spin confinement is more pronounced in that limit than in the 2 ferromagnet setup due to a larger effective spin resistance);

(ii)  $Y \gg X \gg 1$  implying  $m(X, Y, l = 0) \sim \frac{4}{3}(X + 1)$ ;

(iii) finally for  $X \sim Y \gg 1$ ,  $m(X, Y, l = 0) \sim X + 1$  (see Fig. 16-18).

For a realistic value  $X = Y \sim 10$  (see section IV C) this yields an enhancement by a factor

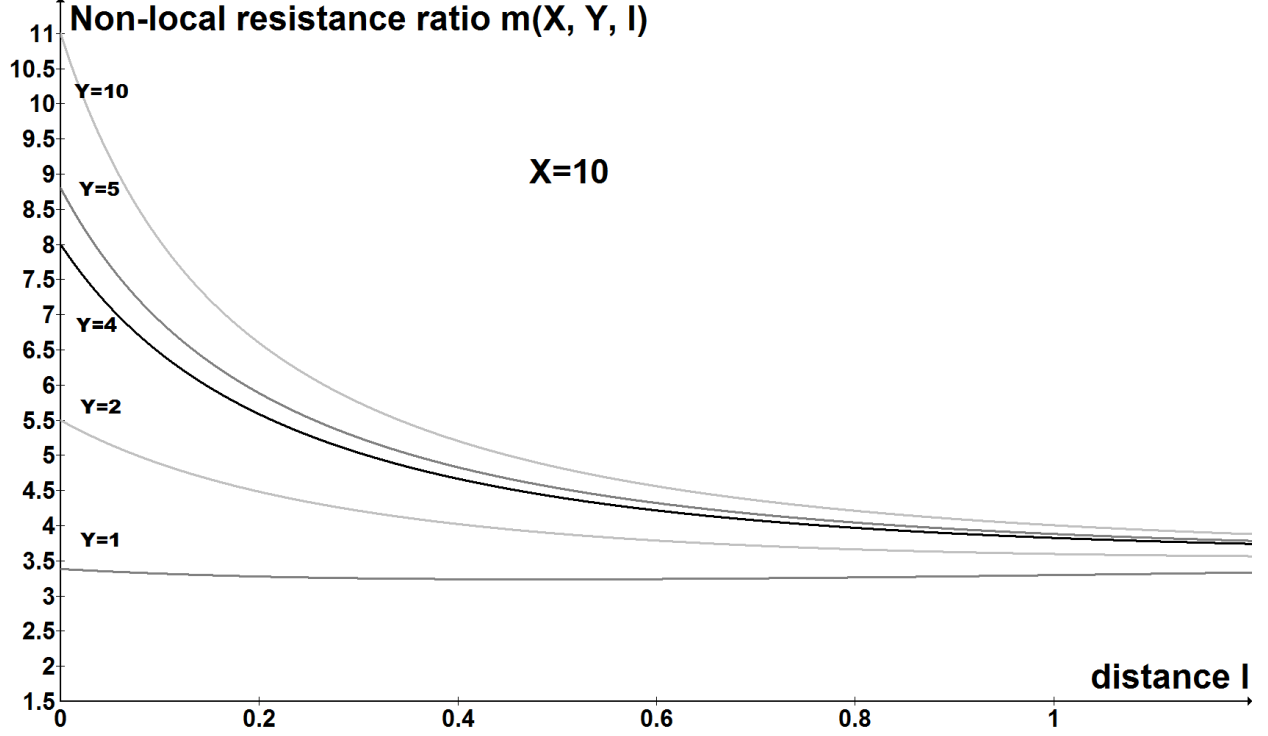


Figure 16. Ratio of non-local resistance in cross geometry (with three ferromagnets) versus lateral setup as a function of distance  $l$  between injector and detector. Spin resistance mismatch is  $X = 10$  at terminals  $F1 - F2 - F3$  and  $Y = 1 - 10$  at paramagnet  $F4$ . All electrodes are in the tunneling regime.

up to 11 reaching therefore an order of magnitude. At a finite distance the enhancement remains considerable ranging from  $m = 3.3 - 6.6$  at  $l = 0.2$  for  $X = 10$  and  $Y = 1 - 10$  (Fig. 16).

If one exchanges  $X$  and  $Y$  (with large  $Y = 10$  and  $X = 1 - 10$ ), one gets weaker signals. This is normal, since when  $Y$  is larger than  $X$ , spin confinement is disfavored since there are then three terminals with spin resistance mismatch smaller than the fourth (Fig. 17).

When  $X$  and  $Y$  are about equal and large,  $X \sim Y \gg 1$  the relative increase ( $\sim X + 1$ ) for a given mismatch  $X$  is smaller than when  $Y \gg X \gg 1$  ( $m \sim \frac{4}{3}(X + 1)$ ); however the latter condition might be more inconvenient to realize for a modest gain (for instance, if we

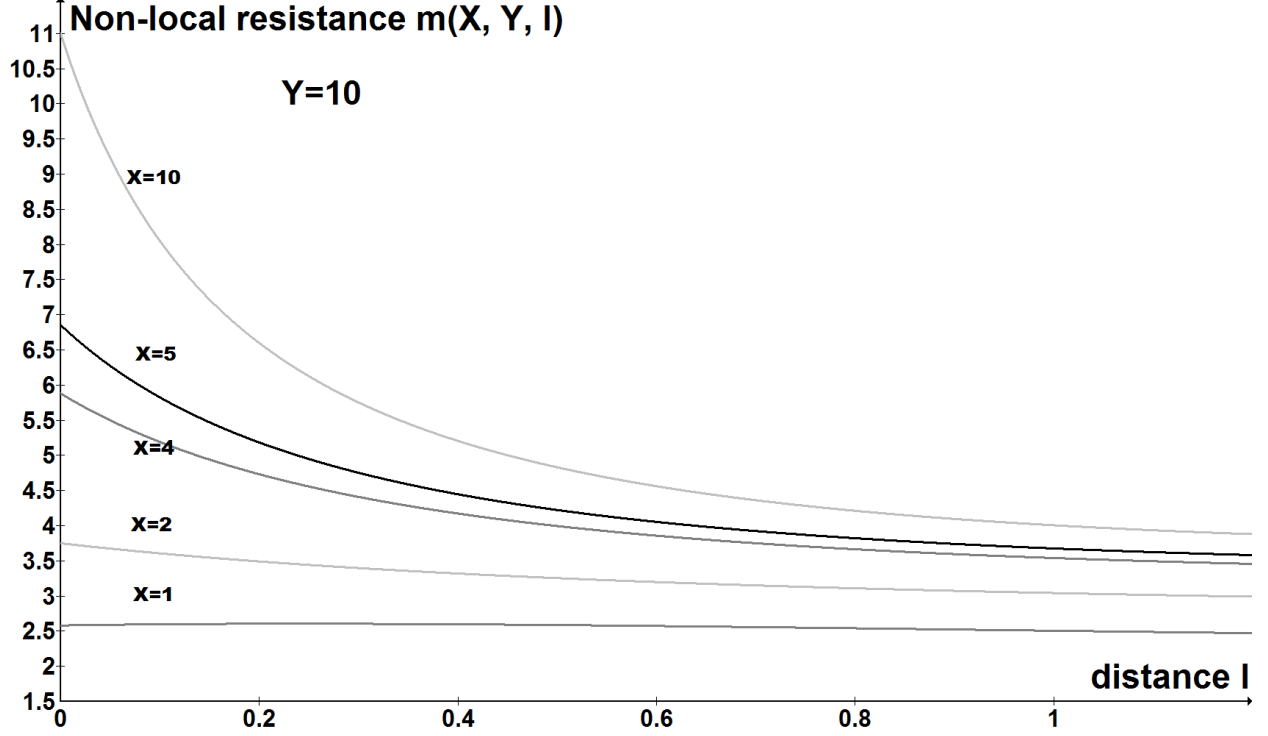


Figure 17. Ratio of non-local resistance in cross geometry (with three ferromagnets) versus lateral setup as a function of distance  $l$  between injector and detector. Spin resistance mismatch is varied ( $X = 1 - 10$ ) at terminals  $F1 - F2 - F3$  and set at  $Y = 10$  at paramagnet  $F4$ . All electrodes are in the tunneling regime..

aim at an order of magnitude increase,  $X \sim 10$  this would imply  $Y \sim 100$  (in the strong tunneling limit) yielding  $m \sim 14$  at  $l = 0$ ; while already for  $X = Y \sim 10$ ,  $m \sim 11$  at  $l = 0$  (see Fig. 18).

In practice the distance between injector and collector can not be reduced arbitrarily. Nevertheless the ratio  $m(X, Y, l)$  may remain large as can be seen in the next figure (Fig. 19) which shows the relative increase of signal at distances  $l = 0.1 - 0.3$ . For  $X \sim Y = 10$ ,  $m \sim 5.7 - 8.0$  for  $l = 0.1 - 0.3$  (which are quite reachable at low temperature for  $l = 0.1$  or even room temperature for  $l = 0.3$ ). This is a doubling of the signal when compared to the values we found for  $X \sim Y = 10$  in the two ferromagnet closed geometry (of section III A 2). (As previously mentioned, this stems from the fact that the total effective polarization is doubled due to the use of two spin injectors while the effective resistances are identical.)

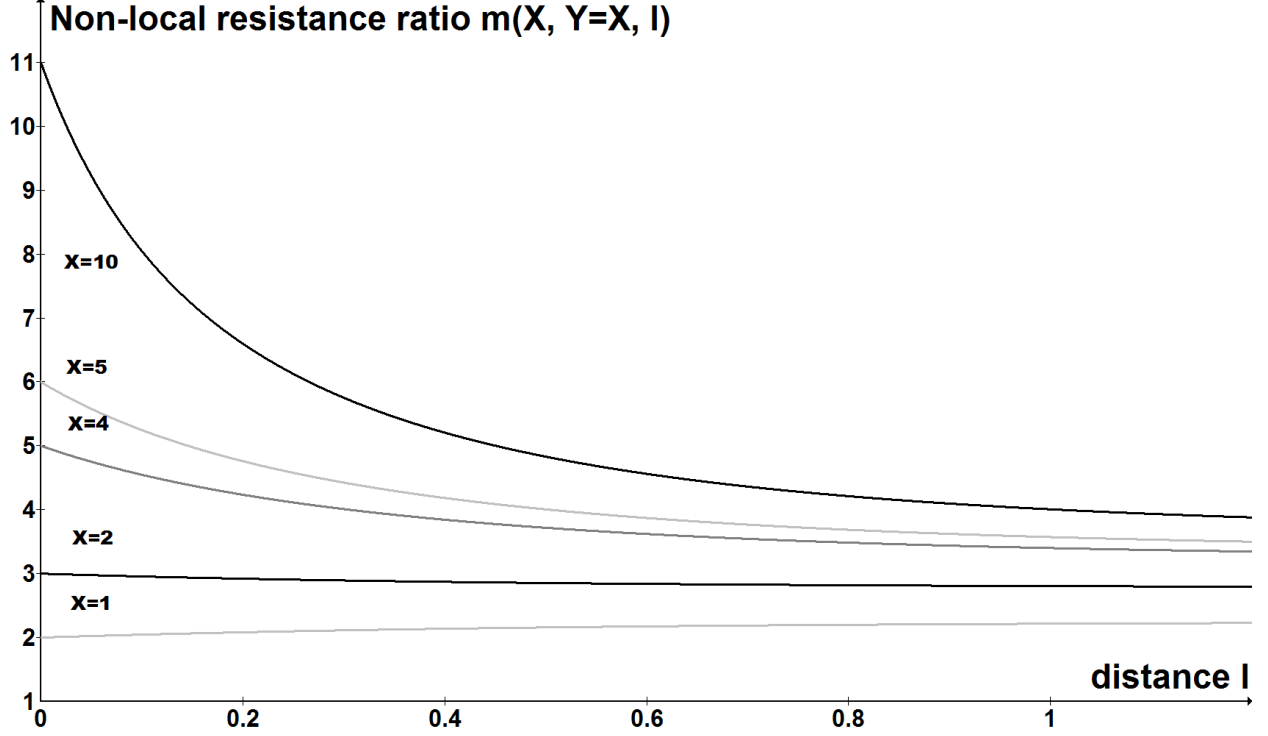


Figure 18. Ratio of non-local resistance in cross geometry (with three ferromagnets) versus lateral setup as a function of distance  $l$  between injector and detector. Spin resistance mismatch is varied  $X = 1 - 10$  at terminals  $F1 - F2 - F3$  but  $Y = X$  at paramagnet  $F4$ . All electrodes are in the tunneling regime..

Turning now to the transparent regime, we still observe an enhancement of the signal (see Fig.20): as before this is entirely due to the doubling of spin injector terminals since there is spin leakage at all terminals. For  $X = Y \leq 1$ , the ratio  $m$  varies between  $X + 1$  and 2 for an increase up to 125 % at large distance.

To summarize this section, we have seen the impact of doubling the number of spin injectors conjugated to spin confinement resulting from large spin resistance mismatches. Although this geometry with three ferromagnets is quite interesting in terms of the large signals which can be achieved, in practice it is better to consider a four ferromagnet setup which will have all the qualities of the 3 ferromagnet Van der Pauw cross with some additional

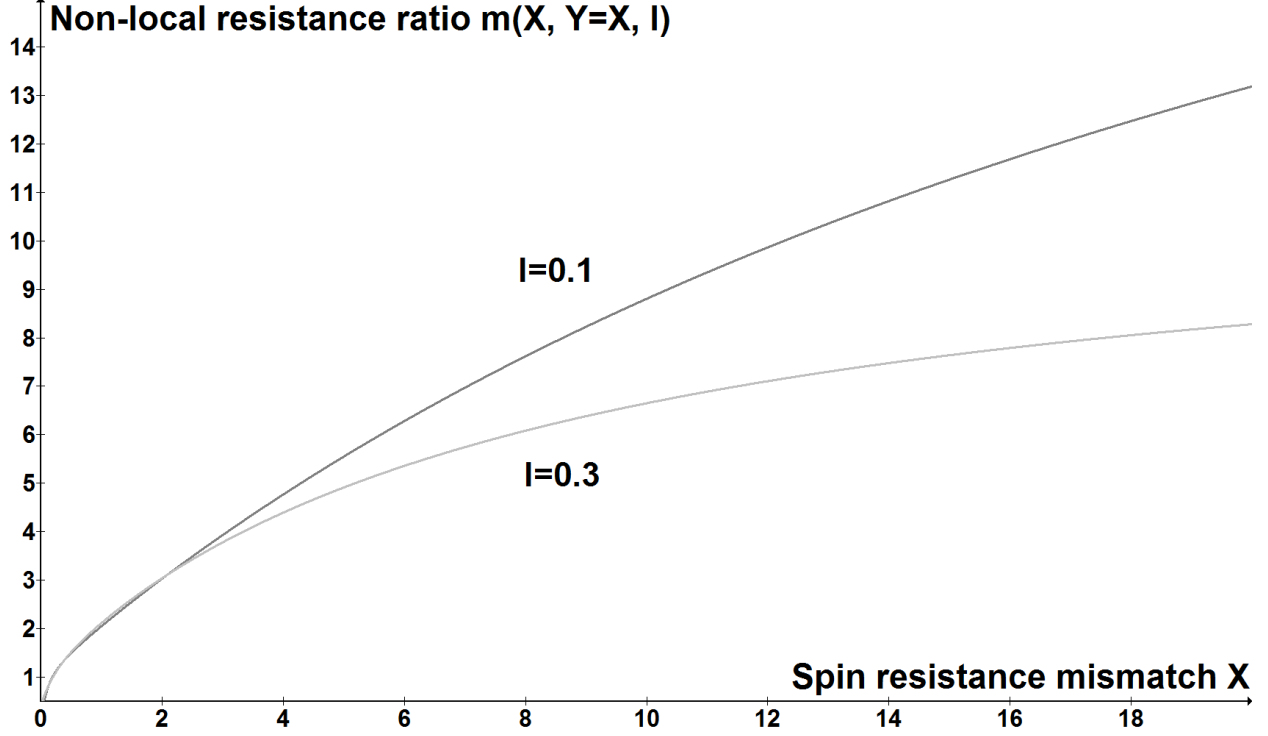


Figure 19. Ratio of non-local resistance in cross geometry (with three ferromagnets) versus lateral setup at distances  $l = 0.1 - 0.3$  between injector and detector. Spin resistance mismatches are set to identical values at all terminals  $X = Y$

properties: protection from voltage offsets (which may be detrimental to SNR) and infinite *GMR* ratio for the non-local resistance.

#### IV. VAN DER PAUW CROSS WITH FOUR FERROMAGNETS.

We now consider now the main device of this paper, a four ferromagnet Van der Pauw setup with the following symmetries: (i) identical injector and collector electrodes ( $F1$  and  $F2$ ), (ii) identical detector electrodes ( $F3$  and  $F4$ ). Besides an enhancement of the spin voltage such a device is protected against voltage offsets and offers an infinite non-local GMR ratio. Other general properties are described in section II C: notably such a setup will also display an ON-OFF switch effect (for the basic 1-bit basic sensing function).

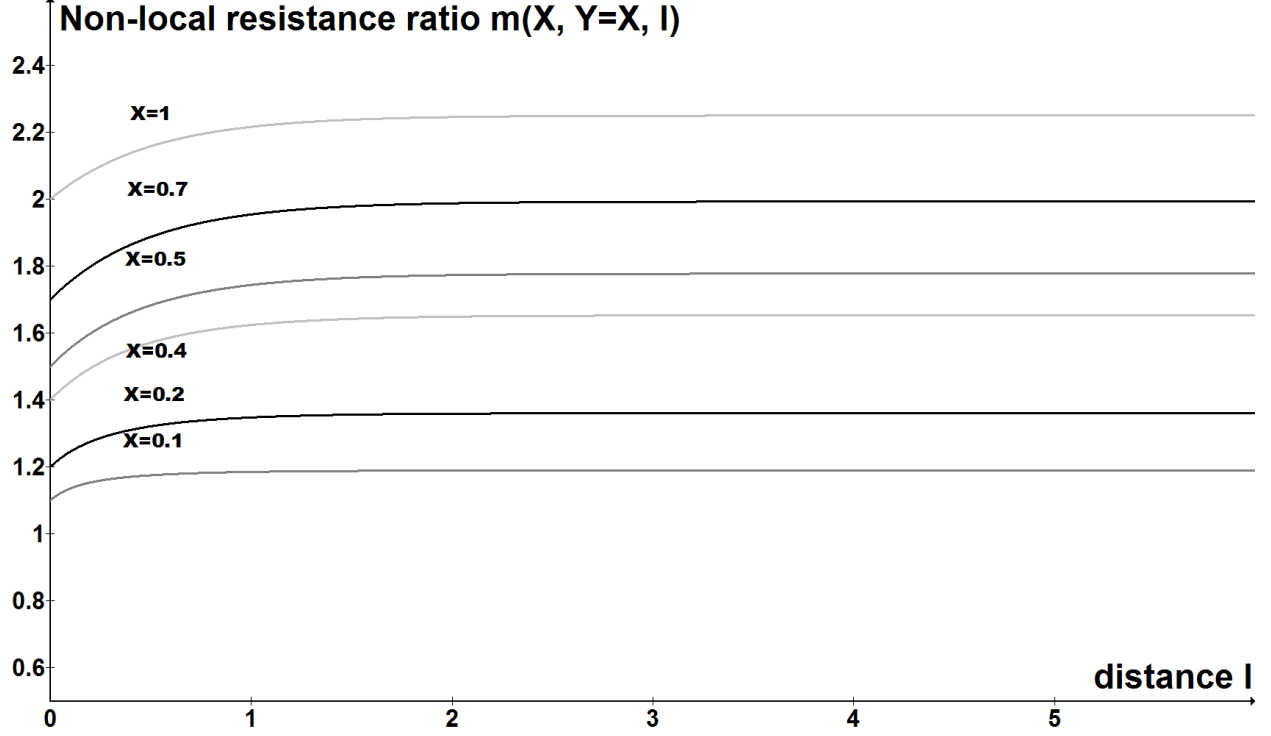


Figure 20. Ratio of non-local resistance in cross geometry (with three ferromagnets) versus lateral setup as a function of distance  $l$  between injector and detector. Spin resistance mismatches are set to identical values at all terminals  $X = Y$  in the transparent regime ( $X = 0.1 - 1$ ).

#### A. Non-local resistance and infinite GMR.

We consider a rhombus geometry for the cross:  $OA = OB = l_1$  while  $OC = OD = l_3$ . The resistance mismatch parameters for the device are set as:

$$X_1 = X_2 = X; X_3 = X_4 = Y \quad (80)$$

while:

$$\widetilde{PR}_1 = \widetilde{PR}_2 = \widetilde{PR}, \quad (81)$$

$$\widetilde{PR}_3 = \widetilde{PR}_4 = \widetilde{PR}'. \quad (82)$$

The non-local resistance [Eq. (23)] becomes:

$$R_{nl} = R_{eff} (\sigma_1 - \sigma_2) (\sigma_3 - \sigma_4) P_{eff,1} P_{eff,3} \quad (83)$$

where  $\sigma_i = \pm 1$  refer to the majority spin direction of terminal  $i$  relative to an absolute axis.

Eventually:

$$R_{nl} = \frac{R_n (\sigma_1 - \sigma_2) (\sigma_3 - \sigma_4) \widetilde{PR} \widetilde{PR}'}{[(X+1)(Y+1)\exp l - (X-1)(Y-1)\exp -l]} \quad (84)$$

where

$$l = l_1 + l_3. \quad (85)$$

Upon magnetization switching of one detector electrode the variation in non-local resistance is therefore:

$$\delta R_{nl} = \frac{4R_n \widetilde{PR} \widetilde{PR}'}{[(X+1)(Y+1)\exp l - (X-1)(Y-1)\exp -l]}$$

### B. Amplitude of non-local resistance.

The signal is largest at small distance where the non-local resistance obeys:

$$\inf(P_c, P_F) \inf(P'_c, P'_F) 4R_n \frac{XY}{(X+Y)} \leq \delta R_{nl}(l \rightarrow 0) \quad (86)$$

with a similar upper bound. Again one recovers the characteristic quadratic dependence on spin resistance mismatches at the numerator (linear at denominator), which imply that if both  $X$  and  $Y$  are large,  $\delta R_{nl}$  will scale with them, much beyond  $R_N$ . If one of the spin mismatches (say  $X$ ) is in the transparent regime, it will set the scale, below or at most at  $R_N$ .

Let us compare in details with the standard lateral setup; we will set  $\widetilde{PR} = \widetilde{PR}'$  and  $X = Y$  for simplicity. (When  $X \neq Y$  the same trends against the case  $X = Y$  are seen as in the three ferromagnet geometry.)

Then the non-local resistance ratio becomes:

$$m(X, l) = \frac{\delta R_{nl}}{\delta R_{nl, lateral}} = \frac{[(2X+1)^2 \exp l - \exp -l]}{[(X+1)^2 \exp l - (X-1)^2 \exp -l]}.$$

At small distance, one gets the enhancement:

$$m(X, l) \rightarrow X + 1. \quad (87)$$

This is the same enhancement as in the 3 ferromagnet setup when  $X = Y$  (see III B 2).

We first consider the tunneling regime ( $X > 1$ ). We plot in the following figures (Fig. (21-22) ) the length and resistance mismatch dependence of the ratio  $m(X, l)$ . For  $X = 1$ ,



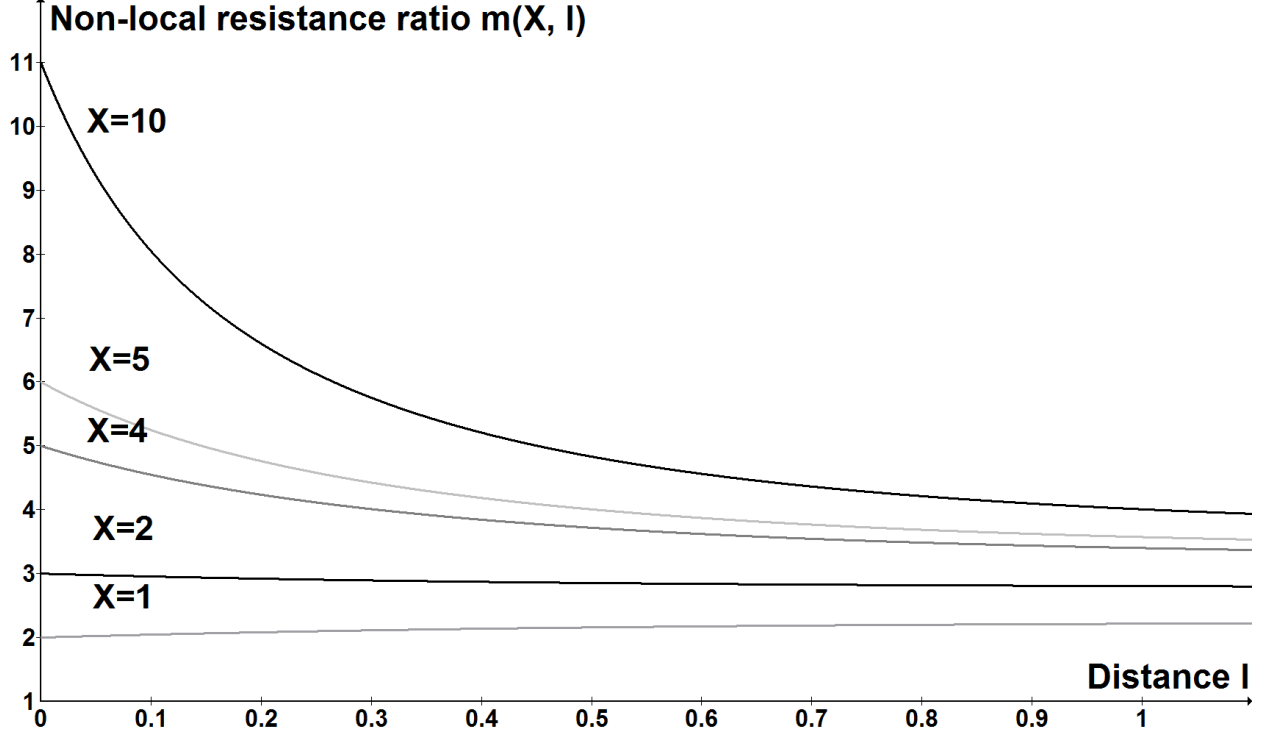


Figure 21. Ratio of non-local resistance in cross setup (with four ferromagnets) versus standard lateral setup, as a function of distance  $l$  between injector and detector. Spin resistance mismatch  $X$  is in the spin confining regime ( $X = 1 - 10$ ).

we observe that  $m \approx 2$  at all distances: in the case of open cross geometry with two ferromagnets (see section III A 1) already one had  $m \sim 1$ ; the doubling here stems therefore from the two spin injectors. For  $X = 10$ , the enhancement remains strong at increasing distance: for instance for  $l = 0.1 - 0.3$ ,  $m \sim 500 - 770 \%$ .

For completeness we now turn to the transparent regime ( $X < 1$ ) although it is less interesting than the tunneling regime if one is to achieve large signals. We observe the same trend in Fig. 23 as before with an increase in several tens of % and up to 100 % at large distance, which stems from the double spin injectors. At shorter distance, the spin current leakage is more pronounced.

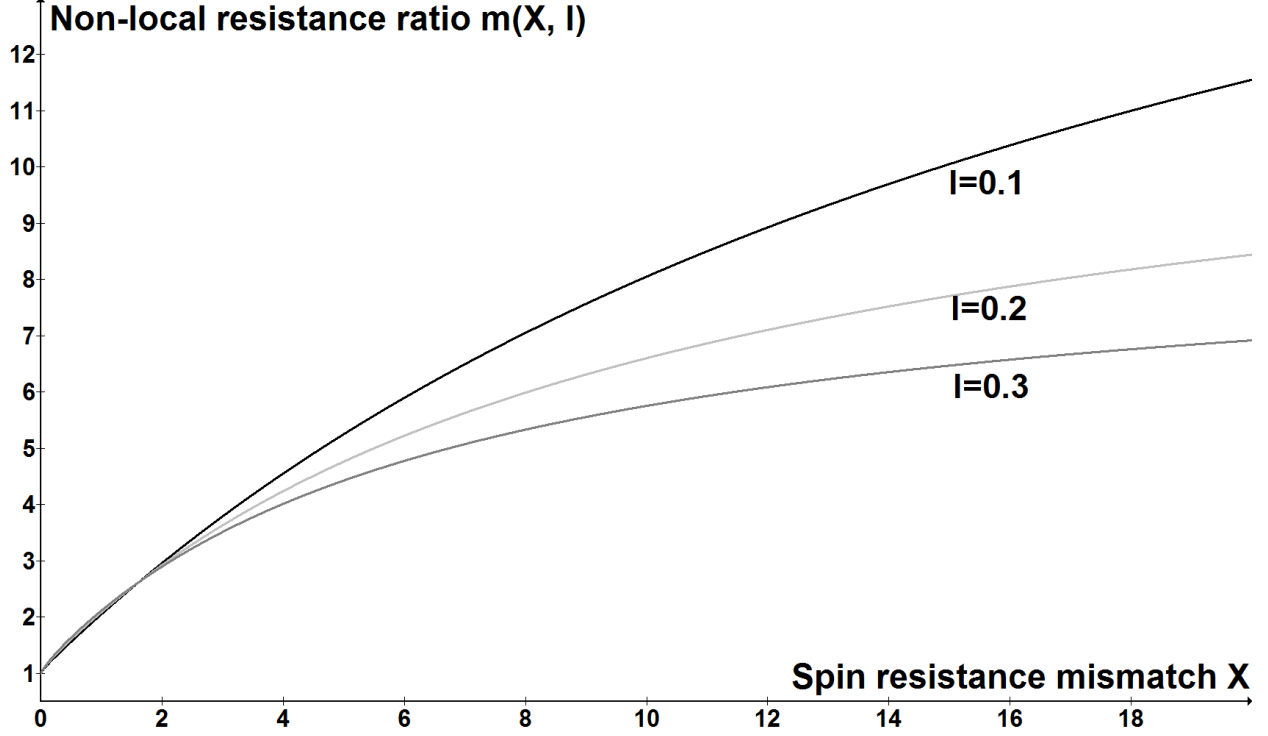


Figure 22. Ratio of non-local resistance in cross setup (with four ferromagnets) versus standard lateral setup, at fixed distance  $l = 0.1 - 0.3$  between injector and detector. Spin resistance mismatch  $X \geq 1$  is in the spin confining regime.

### C. Discussion and comparison to experiments.

Can we expect signals in the  $mV$  range for non-local voltages? Already the state-of-art has reached  $\sim 100 - 200 \mu V$  in lateral spin valves<sup>21</sup>; the peculiarity of these lateral spin valves is that they use thin tunnel  $MgO$  barriers ( $1 - 6 nm$ ) between the ferromagnets (Permalloy  $NiFe$ ) and the paramagnetic channel which is silver  $Ag$ . Due to the thinness of  $MgO$  the interface resistances are much smaller than in usual tunnel junctions, with an interface resistance times cross section product in the range  $R_I A \sim 0.2 \Omega \mu m^2$ . While for as deposited samples the signals are already higher than usual ( $\sim 10 \mu V$ ), after annealing the signals can jump by an order of magnitude; this has been accounted on oxygen vacancies which increase after annealing.

Let us see how much we can get in similar conditions but with the cross geometry with four ferromagnetic electrodes: we will borrow the parameters found of Fukuma and coll.<sup>21</sup> for lateral spin valves based on  $Py - MgO - Ag$  junctions; the size of the junctions is

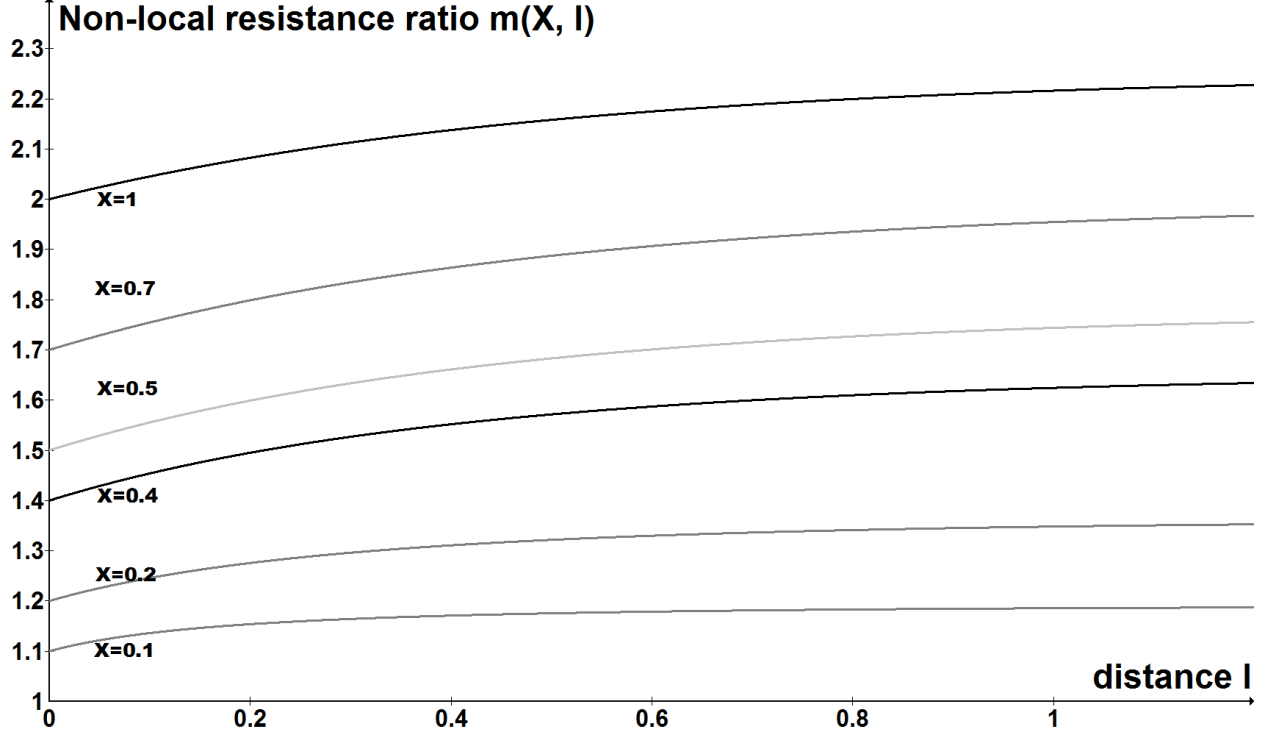


Figure 23. Ratio of non-local resistance in cross setup (with four ferromagnets) versus standard lateral setup, as a function of distance  $l$  between injector and detector. Spin resistance mismatch  $X$  is in the transparent regime ( $X = 0.1 - 1$ ). All terminals are identical in this plot.

$A = 0.022 \mu m^2$ ; the separation between detector and injector is  $L = 300 nm$ .

At 10 K, for a 1 mA current, the largest spin voltage is measured at 112  $\mu V$  with the following parameters:  $P_c = 0.44$  for the interface conductance polarization (noted  $P_I$  by Fukuma and coll.<sup>21</sup>),  $P_F = 0.35$  for *Py* electrodes,  $l_F = 5 nm$ ,  $l_N = 1100 nm$  for the spin relaxation lengths of *Py* and silver *Ag*,  $R_N = \rho_N^* l_N / A = 0.89 \Omega$  for *Ag* spin resistance,  $R_F = \rho_F^* l_F / A = 0.09 \Omega$  ( $R_{NiFe} = 0.08 \Omega$  in Fukuma and coll.<sup>21</sup> is related to our spin resistance  $R_F$  by  $R_{NiFe} = R_F (1 - P_F^2)$ ),  $R_c = 12.13 \Omega$  (our interface spin resistance  $R_c$  is related to  $R_I$  of Fukuma and coll. by  $R_c = R_I / (1 - P_I^2)$  with  $R_I A = 0.2152 \Omega \mu m^2$  and  $A = 0.022 \mu m^2$ ).

Then the spin impedance mismatch is as large as:

$$X = \frac{R_F + R_c}{R_N} = 13.73$$

while:

$$l = L / l_N = 0.27.$$

For these values and using the measured  $\Delta V_{nl} = 110 \mu V$ ,  $\Delta R_{nl} = 110 m\Omega$ ,  $I = 1 mA$  this implies:

$$m(X, l) = 660 \%$$

which would imply a spin voltage variation as large as:

$$\Delta V \sim 725 \mu V$$

or:

$$\Delta R_{nl} \sim 725 m\Omega$$

(since  $I = 1 mA$ ); if we use the symmetric four-terminal cross device, the baseline signal vanishes (for parallel orientation of detector electrodes), so that

$$R A = \Delta R A = 16 m\Omega \mu m^2$$

which compared with CPP GMR spin valves is large in terms of  $\Delta R A$  product (a few  $m\Omega \mu m^2$ ) but small in terms of customary  $R A$  values ( $10 - 100 m\Omega \mu m^2$ )<sup>12</sup>.

Still at 10 K, the highest voltage reported is 200  $\mu V$  for a larger current  $I = 3.5 mA$  so that the non-local resistance is smaller  $\Delta R_{nl} = 63 m\Omega$ ; using our setup we predict one would have achieved signals in the range:

$$\Delta V_{nl} \sim 1.320 mV$$

$$\Delta R_{nl} \sim 415 m\Omega$$

$$\Delta R A \sim 9.1 m\Omega \mu m^2$$

$$j \sim 1.6 \cdot 10^7 A/cm^2.$$

The current density is therefore reasonable although the  $mV$  mark for the spin voltage is reached! The non-local resistance variation is however smaller (Fukuma and coll.<sup>21</sup> attributes the decrease to an enhanced spin relaxation in  $Ag$  due to phonons).

If we put into perspective these predictions, we may remember that the first measurements of non-local resistance in bulk metal wires<sup>1</sup> by Johnson and Silsbee yielded  $\sim n\Omega$ , were thereafter improved in thin films<sup>2</sup> to about 10  $\mu\Omega$ ; Fukuma and coll.<sup>21</sup> have pushed to  $\sim 100 m\Omega$ . The largest non-local resistance variation observed is however already in the Ohm range in lateral tunnel junctions<sup>18,19</sup>, with one major catch: currents must remain small ( $\sim \mu A$ ) due to spin depolarization at larger currents, which results in a spin voltage

still in the customary  $\mu V$  range. In contrast, in metallic lateral spin valves larger currents can be achieved<sup>21,22,49–54</sup>.

Turning now to room temperature, there is few change in spin impedance mismatch  $X$

$$X = 13.44$$

but due to a much smaller  $l_N \sim 300 \text{ nm}$

$$l = 1.$$

One still finds a sizable enhancement:

$$m(X, l) = 415 \%$$

which would lead to the following numbers (using original experimental values of Fukuma and coll.<sup>21</sup>  $\Delta V_{nl} = 51 \mu V$ ,  $\Delta R_{nl} = 51 \text{ m}\Omega$ ,  $I = 1 \text{ mA}$ ):

$$\Delta V_{nl} \sim 210 \mu V$$

$$\Delta R_{nl} \sim 210 \text{ m}\Omega$$

$$\Delta R A \sim 4.5 \text{ m}\Omega \mu m^2$$

$$j \sim 5 \cdot 10^6 \text{ A/cm}^2.$$

There is still room for improvement at room temperature before reaching the  $mV$  mark. One obvious way would be to decrease the distance between injector and detector; for instance suppose  $L = 100 \text{ nm}$  which is quite reasonable in terms of lithography, then  $l = 0.33$ ; taking the previous room-temperature value for  $X$  ( $X = 13.44$ ) would yield:

$$m(X, l) = 605 \%$$

so that

$$\Delta V_{nl} \sim 310 \mu V$$

$$\Delta R_{nl} \sim 310 \text{ m}\Omega$$

$$\Delta R A \sim 6.8 \text{ m}\Omega \mu m^2$$

$$j \sim 5 \cdot 10^6 \text{ A/cm}^2.$$

If we lower to a still achievable  $50 \text{ nm}$  for which  $l = 0.17$  then:

$$m(X, l) = 795 \%$$

and:

$$\Delta V_{nl} \sim 405 \mu V$$

$$\Delta R_{nl} \sim 405 m\Omega$$

$$\Delta R A \sim 8.9 m\Omega\mu m^2$$

$$j \sim 5 \cdot 10^6 A/cm^2.$$

(at such length  $L = 50 nm$   $m = 1150\%$  at  $10K$  so that  $\Delta V_{nl} \sim 1.250 mV$  again in the  $mV$  mark at  $I = 1 mA$ ).

Are there any other ways to further improve the signal at room-temperature? We will show elsewhere<sup>36</sup> that with an all-lateral structure even stronger enhancements are found due to better spin confinement. In the latter example one would get as much as  $m \sim 1070 \%$  at room temperature (or  $m \sim 1320 \%$  at  $10 K$ ).

### **Application to sensor and read-heads technology?**

Reaching the  $1 Tbit/inch^2$  mark up to  $10 Tbit/inch^2$  areal density requires new recording methods to push beyond the super-paramagnetic limit; bit patterning and thermally assisted magnetic recording are currently investigated<sup>13</sup>. But such large areal densities are also very challenging for current sensor technologies. The transition from AMR, CIP GMR, TMR and now CPP GMR has accompanied dramatic and steady increase of the areal density of hard-drives up to about  $0.5 - 0.7 Tbit/inch^2$  (as of late 2011). Further increase to reach the  $1 Tbit/inch^2$  has motivated the latest transition from TMR read-heads to CPP metallic spin valves because TMR is limited to  $R A \sim 1 \Omega\mu m^2$  which will not be small enough to accommodate the high data rates required for the desired areal densities<sup>12</sup>. In contrast CPP GMR read-heads can have  $R A < 0.1 \Omega\mu m^2$  which is the minimum required for an areal density  $1 Tbit/inch^2$ . An additional requirement for CPP spin valves is to have larger  $\Delta R A > 5 m\Omega\mu m^2$  instead of the usual  $< 1 m\Omega\mu m^2$  which are too small to ensure good  $SNR$  (signal to noise ratio). The source of noises are usually Johnson thermal noise in metallic spin valves (both CIP and CPP) or shot noise in TMR valves. But at the shrinking sizes relevant for larger areal densities additional relevant sources of noise appear to be STT (spin transfer torque) noise, mag-noise (noise due to thermal fluctuations in the free layer) and amplifier noise<sup>12,14,55</sup>.

Non-local devices have been barred for practical use due to small signals and have mostly been interesting for fundamental and pedagogical use. As we have shown this may prove less

of a concern by using geometries with better spin confinement and with multiple injectors. Such non-local devices used as sensors would hold several advantages over their standard counterparts while retaining the small  $RA$  of CPP metallic spin valves:

- smaller thickness of the sensing detector (two or three layers since the reference layer is located elsewhere and can have a different coercive field by playing on its geometry);
- diminished sensitivity to STT noise since the largest momentum transfer is between source and drain electrodes;
- no self-field since no current flows through the detectors (Oersted fields are at injectors).
- In the case of a symmetric cross geometry, the possibility to achieve 100 % or maximum MR ratio (using the pessimistic ratio) so that  $R = \Delta R$  is also advantageous regarding Johnson and shot noise providing a 10 – 20  $dB$  boost in SNR (as explained above in section II C).

In addition the maximum MR ratio property allows to satisfy both requirements sought in CPP spin valves: small enough  $RA < 0.1 \Omega\mu m^2$  for high data rates, but large enough  $\Delta RA > 5 m\Omega\mu m^2$  for good SNR. (The latter figure would need to be adjusted to take into account the peculiarities of the non-local device but we use it as a reasonable rule-of-thumb).

The main constraints for a large SNR will remain mag-noise, amplifier noise as well as the ability to scale down the non-local setup to par with the 20 – 30  $nm$  track width relevant<sup>11,14</sup> for areal densities above 1  $Tbit/inch^2$ . Although we expect STT noise to be less of a nuisance, angular momentum transfer still occurs from injectors to detectors and a quantification of critical currents adapted to the geometry would be interesting. The angular response is thus an important aspect to explore in view of a potential use as a sensor and is discussed elsewhere<sup>36</sup>.

Although the geometry studied in this paper might seem quite peculiar, there is actually a lot of flexibility in terms of design, provided the following recommendations are enforced:

- (i) confine spin with large interface resistances (when compared with the paramagnet spin resistance) wherever spin current can leak;
- (ii) in the smallest volume for the paramagnet connecting the ferromagnetic terminals (when compared to  $l_N^3$ );
- (iii) to ensure infinite MR (optimistic) ratio, use two identical ferromagnetic electrodes as detectors within a symmetric arrangement with respect to injectors (the geometry of the ferromagnetic electrodes need not be completely identical: they just need to be identical to

within a few spin diffusion lengths  $l_F$  of the contact to the paramagnet; this is an important practical remark since this allows to have different coercive fields for the various electrodes while keeping the properties stemming from symmetry);

(iv) for further gain in the signal, use two antiparallel ferromagnetic electrodes.

With these prescriptions non-local devices will perhaps be able to reach the realm of applications.

## V. CONCLUSION AND PROSPECTS.

We have presented a variant of non-local spin valves based on a Van der Pauw cross setup: its basic characteristic is its reliance on four collinear ferromagnetic terminals. By playing on the numerous magnetization configurations of the four ferromagnets and on their symmetries several functionalities have been described: (i) an improved non-local spin valve (when used to read 1 bit) with an infinite GMR for the non-local resistance; (ii) ON-OFF switch effect where magnetization at the injectors can control the read-off at detectors; (iii) 3-bit storage or sensing and offset-free 2-bit reading; (iv) direct Spin Hall measurement; (v) use as programmable magneto-logic gates.

In addition to these functionalities the amplitude of the non-local resistance has the potential to be much increased in such a setup due to the conjunction of (i) spin confinement by tunnel barriers; (ii) the use of two electrodes as spin injectors instead of only one. We have also studied the separate or combined impact of both features by considering setups with two or three ferromagnets: the use of two injectors basically doubles the signal while the use of tunnel barriers hinder spin leaking from the Van der Pauw cross, thus increasing spin accumulation within it. The latter effect is already observed even when not all terminals are connected by tunnel barriers to the Van der Pauw cross. An important parameter is the spin resistance mismatch at each ferromagnetic-paramagnetic interface (at the four terminals), which is the ratio between the spin resistances of the ferromagnet and the paramagnet. The larger this parameter, the better the spin confinement in the Van der Pauw cross.

Additional functionalities will be discussed elsewhere: non-collinear properties which are essential for sensor applications; as well as caloritronic ones. A related non-local device in the lateral geometry for which signals are much stronger will be also investigated<sup>36</sup>.



## Appendix A: Computing the Spin Voltage.

We derive in this Appendix the general expression of the spin voltage by relying on the one-dimensional drift-diffusion equations<sup>7,31,32</sup>.

### 1. Basic equations.

#### *a. Spin accumulation vectors.*

We orient spin currents away from the origin  $O$  of the cross.

In each electrode ( $F1 - F4$ ) the spin accumulation and spin currents are then:

$F1$ :

$$\Delta\mu_{F1}(z) = \Delta\mu_{F1}(A) \exp\left(-\frac{z - L_1}{l_{F1}}\right) \quad (\text{A1})$$

$$I_{s,F1}(z) = -P_{F1}I_c - \frac{\Delta\mu_{F1}(A)}{R_{F1}} \exp\left(-\frac{z - L_1}{l_{F1}}\right) \quad (\text{A2})$$

so that:

$$I_{s,F1}(A) = -P_{F1}I_c - \frac{\Delta\mu_{F1}(A)}{R_{F1}}. \quad (\text{A3})$$

$F2$ : without loss of generality we assume  $F2$  antiparallel to  $F1$  which allows to have symmetric equations (for the parallel case one should take opposite polarizations)

$$I_{s,F2}(B) = -P_{F2}I_c - \frac{\Delta\mu_{F2}(B)}{R_{F2}} \quad (\text{A4})$$

where  $I_{s,F2}(B)$  is counted positive when flowing in the direction  $z' = -z$ ;

$F3$ :

$$I_{s,F3}(C) = -\frac{\Delta\mu_{F3}(C)}{R_{F3}} \quad (\text{A5})$$

where  $I_{s,F3}(B)$  is oriented in the direction  $x$ .

$F4$ :

$$I_{s,F4}(D) = -\frac{\Delta\mu_{F4}(D)}{R_{D4}} \quad (\text{A6})$$

where  $I_{s,N4}(D)$  is oriented in the direction  $x' = -x$ .

In the arms ( $I - IV$ ) we define spin accumulation vectors which are related to the spin accumulation and spin currents by:

$I$ :

$$\Delta\mu_I(z) = \mathbf{K}_I \cdot \left( \exp \frac{z}{l_N}; \exp -\frac{z}{l_N} \right), \quad (\text{A7})$$

$$I_{s,I}(z) = \frac{\mathbf{K}_I}{R_N} \cdot \left( \exp \frac{z}{l_N}; -\exp -\frac{z}{l_N} \right) \quad (\text{A8})$$

where  $\mathbf{K}_I$  is a constant two-component vector.

II: following our convention of orienting away from the origin

$$\Delta\mu_{II}(z') = \mathbf{K}_{II} \cdot \left( \exp \frac{z'}{l_N}; \exp -\frac{z'}{l_N} \right), \quad (\text{A9})$$

$$I_{s,II}(z') = \frac{\mathbf{K}_{II}}{R_N} \cdot \left( \exp \frac{z'}{l_N}; -\exp -\frac{z'}{l_N} \right) \quad (\text{A10})$$

III:

$$\Delta\mu_{III}(x) = \mathbf{K}_{III} \cdot \left( \exp \frac{x}{l_N}; \exp -\frac{x}{l_N} \right), \quad (\text{A11})$$

$$I_{s,III}(x) = \frac{\mathbf{K}_{III}}{R_N} \cdot \left( \exp \frac{x}{l_N}; -\exp -\frac{x}{l_N} \right) \quad (\text{A12})$$

IV:

$$\Delta\mu_{IV}(x') = \mathbf{K}_{IV} \cdot \left( \exp \frac{x'}{l_N}; \exp -\frac{x'}{l_N} \right), \quad (\text{A13})$$

$$I_{s,IV}(x') = \frac{\mathbf{K}_{IV}}{R_N} \cdot \left( \exp \frac{x'}{l_N}; -\exp -\frac{x'}{l_N} \right) \quad (\text{A14})$$

*b. Conditions at ferromagnetic - paramagnetic interfaces and at cross center.*

**Interfaces.** At interfaces we will neglect all spin flip but assume interface resistance is spin dependent. This implies that spin current at points  $A - D$  is continuous:

$$I_{s,F1}(A) = I_{s,I}(A) \quad (\text{A15})$$

and so forth at points  $B - D$ .

At points  $A - D$  the interface resistance induces a discontinuity in spin accumulations:

$$\Delta\mu_{F1}(A) - \Delta\mu_I(A) = R_{c1} I_s(A) + R_{c1} P_{c1} I_c \quad (\text{A16})$$

$$\Delta\mu_{F2}(B) - \Delta\mu_{II}(B) = R_{c2} I_s(B) + R_{c2} P_{c2} I_c \quad (\text{A17})$$

$$\Delta\mu_{F3}(C) - \Delta\mu_{III}(C) = R_{c3} I_s(C) \quad (\text{A18})$$

$$\Delta\mu_{F4}(D) - \Delta\mu_{IV}(D) = R_{c4} I_s(D) \quad (\text{A19})$$

**Cross center.** At  $O$ , the spin accumulations are continuous:

$$\Delta\mu_I(O) = \Delta\mu_{II}(B) = \Delta\mu_{III}(C) = \Delta\mu_{IV}(O) \quad (\text{A20})$$

and spin current continuity implies:

$$0 = I_{s,I}(O) + I_{s,II}(O) + I_{s,III}(O) + I_{s,IV}(O) \quad (\text{A21})$$

so that for  $\alpha = I - IV$ :

$$\mathbf{K}_\alpha \cdot (1; 1) \equiv \Delta\mu(O); \quad (\text{A22})$$

$$\sum_{\alpha} \mathbf{K}_\alpha \cdot (1; -1) = 0. \quad (\text{A23})$$

## 2. Solution of the equations.

### a. Spin accumulation vectors $\mathbf{K}_\alpha$

We first solve for all four spin accumulation vectors in the arms ( $I - IV$ ).

From the equations (A3, A15, A16) at interface  $F1 - I$  one gets:

$$\Delta\mu_I(A) + (R_{c1} + R_{F1}) I_s(A) = -(P_{F1}R_{F1} + P_{c1}R_{c1}) I_c \quad (\text{A24})$$

After substitution in terms of spin accumulation vector  $\mathbf{K}_I$ :

$$\begin{aligned} & \mathbf{K}_I \cdot ((1 + X_1) \exp l_1; (1 - X_1) \exp -l_1) \\ &= - (P_{F1}R_{F1} + P_{c1}R_{c1}) I_c \end{aligned} \quad (\text{A25})$$

where:

$$X_1 = \frac{R_{F1} + R_{c1}}{R_N} \quad (\text{A26})$$

and

$$l_1 = \frac{L_1}{l_N}. \quad (\text{A27})$$

$\mathbf{K}_{\text{II}}$  obeys a similar equation by changing the index 1 with index 2 since we have assumed that the magnetization of  $F2$  is antiparallel to that of  $F1$ .

$$\begin{aligned} \mathbf{K}_{\text{II}} \cdot ((1 + X_2) \exp l_2; (1 - X_2) \exp -l_2) \\ = - (P_{F2}R_{F2} + P_{c2}R_{c2}) I_c \end{aligned} \quad (\text{A28})$$

while the equations for arms  $III - IV$  obtain by cancelling the charge current:

$$\mathbf{K}_{\text{III}} \cdot ((1 + X_3) \exp l_3; (1 - X_3) \exp -l_3) = 0, \quad (\text{A29})$$

$$\mathbf{K}_{\text{IV}} \cdot ((1 + X_4) \exp l_4; (1 - X_4) \exp -l_4) = 0. \quad (\text{A30})$$

with  $X_{2/3/4}$  and  $l_{2/3/4}$  defined as  $X_1$  and  $l_1$ .

Using the continuity of spin accumulation and spin current at  $O$  yields a second equation for each spin accumulation vector:

$$\mathbf{K}_\alpha \cdot (1; 1) = \Delta\mu(O), \quad (\alpha = \text{I}, \dots, \text{IV}) \quad (\text{A31})$$

Solving for the  $\mathbf{K}_\alpha$  vectors in terms of  $\Delta\mu(O)$  leads to the following expressions:

$$\begin{aligned} \mathbf{K}_{\text{I}} = \frac{\Delta\mu(O)}{2\delta_1^+} \begin{pmatrix} (X_1 - 1) \exp -l_1 \\ (X_1 + 1) \exp l_1 \end{pmatrix} \\ + \frac{(P_{F1}R_{F1} + P_{c1}R_{c1})}{2\delta_1^+} I_c \begin{pmatrix} -1 \\ 1 \end{pmatrix} \end{aligned} \quad (\text{A32})$$

$$\mathbf{K}_{\text{II}} = 1 \longleftrightarrow 2 \quad (\text{A33})$$

while:

$$\mathbf{K}_{\text{III}} = \frac{\Delta\mu(O)}{2\delta_3^+} \begin{pmatrix} (X_3 - 1) \exp -l_3 \\ (X_3 + 1) \exp l_3 \end{pmatrix}, \quad (\text{A34})$$

$$\mathbf{K}_{\text{IV}} = 3 \longleftrightarrow 4, \quad (\text{A35})$$

where:

$$\delta_i^\pm = \frac{(X_i + 1)}{2} \exp l_i \pm \frac{(X_i - 1)}{2} \exp -l_i. \quad (\text{A36})$$

*b. Spin currents at cross center.*

The relation between spin currents at  $O$  on each arm and the spin accumulation  $\Delta\mu(O)$  is derived using:

$$I_{s,I}(O) = \frac{1}{R_N} \mathbf{K}_I \cdot (1; -1) \quad (\text{A37})$$

so that:

$$I_{s,I}(O) = -\frac{\Delta\mu(O)}{R_{eff,1}} - P_{eff,1} I_c \quad (\text{A38})$$

with a similar relation for arm  $II$  where:

$$R_{eff,i} = R_N \frac{\delta_i^+}{\delta_i^-} \quad (\text{A39})$$

$$P_{eff,i} = \frac{\widetilde{PR}_i}{\delta_i^+} \quad (\text{A40})$$

and:

$$\widetilde{PR}_i = [P_{Fi}R_{Fi} + P_{ci}R_{ci}] / R_N \quad (\text{A41})$$

For arm  $III$ , one has simply:

$$I_{s,III}(O) = -\frac{\Delta\mu(O)}{R_{eff,3}} \quad (\text{A42})$$

and a similar relation for arm  $IV$ .

*c. Spin accumulation at cross center.*

Plugging the expressions of the spin currents into the spin current continuity equation at  $O$  [Eq. (A21)] allows determination of  $\Delta\mu(O)$ :

$$\Delta\mu(O) = -(P_{eff,1} + P_{eff,2}) \frac{1}{\sum_{i=1-4} R_{eff,i}^{-1}} I_c. \quad (\text{A43})$$

Observe that the expression of  $\Delta\mu(O)$  does not depend on the polarizations of the ferromagnetic electrodes used as detectors ( $F_3$  and  $F_4$ ): it is fully determined by the injectors

only ( $F1$  and  $F2$ ). The previous expression has been derived by assuming  $F1$  and  $F2$  antiparallel, which reflects into the polarizations signs. In the more general case of arbitrary collinear orientation, one has:

$$\Delta\mu(O) = -(P_{eff,1} - P_{eff,2}) R_{eff} I_c \quad (\text{A44})$$

where positive polarizations are defined relative to an absolute axis (negative polarizations in the opposite direction). The largest spin accumulation in magnitude is achieved for antiparallel injector and collector electrodes.

This can be plugged in the expressions for the spin current:

$$I_{s,I}(O) = \left[ \frac{(P_{eff,1} - P_{eff,2}) R_{eff}}{R_{eff,1}} - P_{eff,1} \right] I_c \quad (\text{A45})$$

with a similar expression for arm  $II$  while for arm  $III$  (or  $IV$  with appropriate substitutions):

$$I_{s,III}(O) = \left[ \frac{(P_{eff,1} - P_{eff,2}) R_{eff}}{R_{eff,3}} \right] I_c. \quad (\text{A46})$$

#### *d. Spin voltage.*

The voltage is measured as:

$$V_{nl} = -[\mu_{F3}(+\infty) - \mu_{F4}(+\infty)]/e. \quad (\text{A47})$$

This voltage depends on the orientations of the four ferromagnetic terminals  $F1 - F4$  so that in the most general case it can assume sixteen distinct values ( $2^4$ ); but since the setup is invariant under a joint flipping of all the magnetizations this number is reduced to eight.

We will need the chemical potential variation at interfaces:

$$\begin{aligned} \mu_{F3}(C) - \mu_{III}(C) &= -R_{c3} P_{c3} I_s(C); \\ \mu_{IV}(D) - \mu_{F4}(D) &= R_{c4} P_{c4} I_s(D). \end{aligned}$$

Leaving out the factor  $e$ :

$$\begin{aligned} -V_{nl} &= [\mu_{F3}(+\infty) - \mu_{F3}(C)] + [\mu_{F3}(C) - \mu_{III}(C)] \\ &\quad + [\mu_{III}(C) - \mu_{III}(O)] + [\mu_{IV}(O) - \mu_{IV}(D)] \\ &\quad + [\mu_{IV}(D) - \mu_{F4}(D)] + [\mu_{F4}(D) - \mu_{F4}(+\infty)] \\ &= P_{F3} \Delta\mu_{F3}(C) - R_{c3} P_{c3} I_s(C) \\ &\quad - P_{F4} \Delta\mu_{F4}(D) + R_{c4} P_{c4} I_s(D) \end{aligned}$$

so that:

$$\begin{aligned}
-V_{nl} &= \left[ \frac{P_{F3}R_{F3} + P_{c3}R_{c3}}{R_{F3}} \Delta\mu_{F3}(C) \right] \\
&+ \left[ \frac{P_{F4}R_{F4} + P_{c4}R_{c4}}{R_{F4}} \Delta\mu_{F4}(D) \right] \\
&= \left[ \frac{P_{F3}R_{F3} + P_{c3}R_{c3}}{R_{F3} + R_{c3}} \Delta\mu_{III}(C) \right] \\
&- \left[ \frac{P_{F4}R_{F4} + P_{c4}R_{c4}}{R_{F4} + R_{c4}} \Delta\mu_{IV}(D) \right]
\end{aligned}$$

The spin accumulations  $\Delta\mu_{III}(C)$  and  $\Delta\mu_{IV}(D)$  are derived easily from the spin accumulation vectors  $\mathbf{K}_{III}$  and  $\mathbf{K}_{IV}$  [Eq. (A34-A35)]:

$$\begin{aligned}
\Delta\mu_{III}(C) &= \mathbf{K}_{III} \cdot (\exp l_3; \exp -l_3) \\
&= \frac{\Delta\mu(O)}{\delta_3^+} X_3
\end{aligned} \tag{A48}$$

and:

$$\Delta\mu_{IV}(D) = 3 \longleftrightarrow 4$$

One thus gets a voltage drop:

$$V_{nl} = -\Delta\mu(O) \left( \frac{\widetilde{PR}_3}{\delta_3^+} - \frac{\widetilde{PR}_4}{\delta_4^+} \right) \tag{A49}$$

$$= -\Delta\mu(O) (P_{eff,3} - P_{eff,4}) \tag{A50}$$

*e. Spin currents and spin accumulations at detector terminals.*

The spin current at point  $C$  is given by:

$$I_s(C) = \frac{1}{R_N} \mathbf{K}_{III} \cdot (\exp l_3; -\exp -l_3) \tag{A51}$$

so that:

$$I_s(C) = -\frac{\Delta\mu(O)}{R_N \delta_3^+} = \frac{(P_{eff,1} - P_{eff,2}) R_{eff} I_c}{R_N \delta_3^+}. \tag{A52}$$

Let us study  $|I_s(C)|$  as a function of  $X_3$  while keeping all other parameters fixed.

$$\begin{aligned}
|I_s(C)| &\propto \frac{1}{a\delta_3^+ + \delta_3^-} \\
&\propto \frac{1}{2 \{X_3 [a \cosh l_3 + \sinh l_3] + [a \sinh l_3 + \cosh l_3]\}}
\end{aligned}$$

where:

$$a = \sum_{i \neq 3} R_{eff,i}^{-1} > 0.$$

The spin current is therefore (in magnitude) a decreasing function of  $X_3$ : in order to have large spin currents it is therefore better to have transparent junctions rather than large tunnel barriers.

The spin accumulation at  $C$  behaves in an opposite manner:

$$\begin{aligned} |\Delta\mu_{III}(C)| &= \frac{|\Delta\mu(O)|}{\delta_3^+} X_3 \\ &\propto \frac{X_3}{a\delta_3^+ + \delta_3^-} \\ &\propto \frac{X_3}{X_3 [a \cosh l_3 + \sinh l_3] + [a \sinh l_3 + \cosh l_3]} \end{aligned}$$

which is an increasing function of  $X_3$ . A large accumulation at the interface with detector  $F3$  is favored by large spin impedance mismatch  $X_3$ .

The spin currents and accumulations at source and drain can be studied in the same manner with identical conclusions: large  $X$  leads to larger spin accumulations but smaller spin currents; with an opposite result for small  $X$ .

### 3. Local resistance and non-local charge current.

The resistance can also be measured locally along the charge current path from source to drain; if additionally we close the circuit between the two detector electrodes a charge current is generated.

#### *a. Local resistance at injectors.*

The voltage probes at source and drain are positioned at  $z_1$  and  $z_2$  which we will assume to be very large ( $\gg l_{Fi}$  the spin relaxation lengths in the ferromagnets).

$$V_{local} = - [\mu_{F1}(z_1) - \mu_{F2}(z_2)]. \quad (\text{A53})$$

(we have left out the factor  $e$ ).

We will need the dependence of the chemical potentials as well as the interface discontinuities:



$$\mu_{F1}(z) = -\rho_{F1}^* (1 - P_{F1}^2) I_c z + C_1 - P_{F1} \Delta \mu_{F1}(z) \quad (\text{A54})$$

$$\mu_{F2}(z') = \rho_{F2}^* (1 - P_{F2}^2) I_c z' + C_2 - P_{F2} \Delta \mu_{F2}(z') \quad (\text{A55})$$

where  $C_{1-2}$  are constants and:

$$\mu_{F1}(A) - \mu_I(A) = -R_{c1} (I_c + P_{c1} I_s(A)) \quad (\text{A56})$$

$$\mu_{F2}(B) - \mu_{II}(B) = R_{c2} (I_c - P_{c2} I_s(B)) \quad (\text{A57})$$

Since:

$$\begin{aligned} -V_{local} &= [\mu_{F1}(z_1) - \mu_{F1}(A)] + [\mu_{F1}(A) - \mu_I(A)] \\ &\quad + [\mu_I(A) - \mu_{II}(B)] + [\mu_{II}(B) - \mu_{F2}(B)] \\ &\quad + [\mu_{F2}(B) - \mu_{F2}(z_2)] \end{aligned} \quad (\text{A58})$$

eventually:

$$\begin{aligned} -V_{local} &= \left[ \frac{P_{F1} R_{F1} + P_{c1} R_{c1}}{R_{F1} + R_{c1}} \Delta \mu_I(A) \right] \\ &\quad - \left[ \frac{P_{F2} R_{F2} + P_{c2} R_{c2}}{R_{F2} + R_{c2}} \Delta \mu_{II}(B) \right] \\ &\quad - R_0 I_c \end{aligned} \quad (\text{A59})$$

where  $R_0(\{P_{1-2}\})$  is an even function of polarizations  $\{P_{1-2}\} = P_{Fi}, P_{ci}, i = 1 - 2$ :

$$\begin{aligned} R_0 \{P_{1-2}\} &= \sum_{i=1-2} \rho_{Fi}^* (1 - P_{Fi}^2) z_i + \rho_N^* l_i \\ &\quad + R_{ci} + \frac{[P_{Fi}^2 R_{ci} R_{Fi} - P_{ci}^2 R_{ci}^2 - 2P_{Fi} P_{ci} R_{Fi} R_{ci}]}{R_{Fi} + R_{ci}}. \end{aligned}$$

(It can be checked that  $R_0 \geq 0$  as it should be.)

Since:

$$\Delta \mu_I(A) = \mathbf{K}_I \cdot (\exp l_1; \exp -l_1) = \frac{\Delta \mu(O)}{\delta_1^+} X_1 \quad (\text{A60})$$

(with a similar expression for  $\Delta \mu_{II}(B)$ )

$$\begin{aligned} -V_{local} &= \Delta \mu(O) [P_{eff,1} - P_{eff,2}] - R_0 I_c \\ &= -R_{eff} [P_{eff,1} - P_{eff,2}]^2 I_c - R_0 I_c \end{aligned} \quad (\text{A61})$$

The resistance variation when one flips terminal  $F2$  is therefore:

$$\begin{aligned}\Delta R_{local} &= R_{AP} - R_P = \frac{V_{AP} - V_P}{I_c} \\ &= 4 P_{eff,1} P_{eff,2} R_{eff}\end{aligned}\tag{A62}$$

*b. Local resistance at detectors and non-local charge current.*

The previous expressions will help us to derive the expression of the non-local charge current flowing through the detectors  $F3$  and  $F4$  when they are in closed circuit instead of being in open circuit.

Suppose one drives a current  $I'_c$  through terminals  $F3$  and  $F4$  but not through  $F1$  and  $F2$  exchanging the roles of injectors and detectors ( $I_c = 0$ );

$$V_{34} = R_{eff} [P_{eff,3} - P_{eff,4}]^2 I'_c + R_0 \{P_{3-4}\} I'_c\tag{A63}$$

We can define the local conductance at detectors:

$$I'_c = g_{34} V_{34}\tag{A64}$$

where

$$g_{34} = [R_{eff} [P_{eff,3} - P_{eff,4}]^2 + R_0 \{P_{3-4}\}]^{-1}.\tag{A65}$$

We now switch on the charge current  $I_c$  at source and drain  $F1$  and  $F2$ ; when  $I'_c = 0$ ,

$$\begin{aligned}V_{34} &= V_{nl} = R_{nl} I_c \\ &= R_{eff} (P_{eff,1} - P_{eff,2}) (P_{eff,3} - P_{eff,4}) I_c.\end{aligned}\tag{A66}$$

When both  $I_c$  and  $I'_c$  are switched on, by superposition one gets the general expression of the current flowing through detector electrodes:

$$I'_c = g_{34} (V_{34} - R_{nl} I_c).\tag{A67}$$

If we short  $F3$  and  $F4$ ,  $V_{34} = 0$  so that the non-local charge current is:

$$I_{nl} = -g_{34} R_{nl} I_c\tag{A68}$$

or:

$$I_{nl} = -\frac{R_{eff} (P_{eff,1} - P_{eff,2}) (P_{eff,3} - P_{eff,4})}{R_{eff} [P_{eff,3} - P_{eff,4}]^2 + R_0 \{P_{3-4}\}} I_c\tag{A69}$$

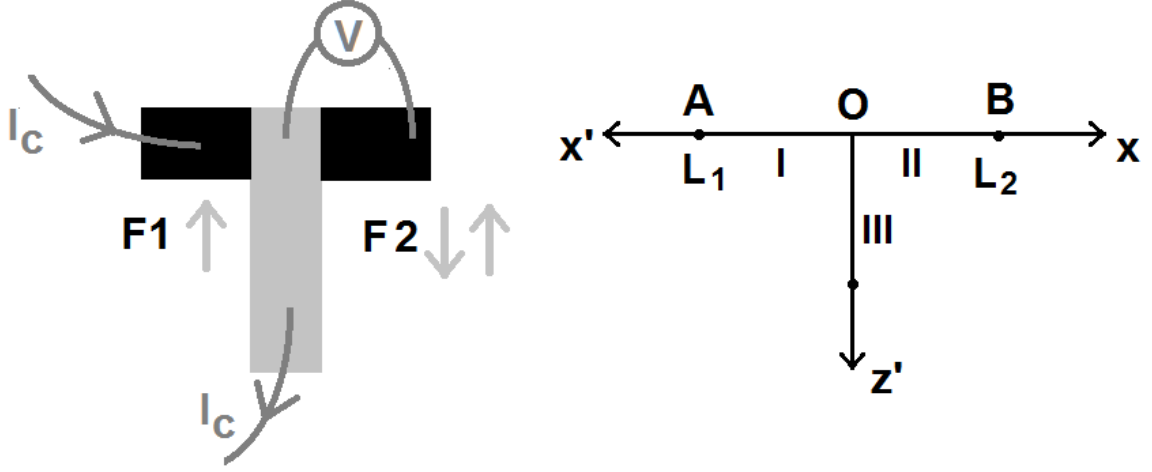


Figure 24. Left: geometry used by Hershfield-Zhao and Fert-Lee for the bipolar spin switch calculations. Right: geometry in our simplified one-dimensional treatment which includes spin leaking to the bottom.

### Appendix B: Revisiting the bipolar spin switch.

The original theory by Johnson of the bipolar spin switch was refined by Hershfield and Zhao<sup>48</sup> who included spin relaxation in the ferromagnets, and also by Fert and Lee<sup>47</sup> who studied the (adverse) impact of interface spin flips. If we discard the effect of spin flips, the non-local resistance variation was found by Hershfield and Zhao to be:

$$\Delta R_{nl} = \frac{4R_n \widetilde{PR} \widetilde{PR}'}{[(X+1)(Y+1)\exp l - (X-1)(Y-1)\exp -l]} \quad (\text{B1})$$

where we have renamed parameters according to the definitions used in this paper:  $X$  and  $\widetilde{PR} = (P_c R_c + P_F R_F)/R_N$  are defined at injector while  $Y$  and  $\widetilde{PR}'$  pertain to the detector. Fert and Lee have the same expression in the case  $\widetilde{PR} = \widetilde{PR}'$  and  $X = Y$  (identical ferromagnets at injector and detector).

What is noteworthy is that this expression is actually identical to that we found for the four ferromagnetic terminal cross geometry. In the limit of large  $X$  and  $Y$ ,  $\Delta R_{nl}$  might therefore be quite large as has been discussed repeatedly in this paper due to spin confinement.

But a look at the geometry may cast doubt on this result (see Fig. 24): there should be some spin leakage in the setup even if both injector and detector are in tunnel contacts with the central paramagnet, because spin is leaving along with charge on the bottom

paramagnetic arm used as (charge) current drain. Hershfield and Zhao argued that spin leakage could be neglected in the bottom arm since in Johnson's original experiment the *voltage* pads are much larger than the spin relaxation length (the pads area was  $0.01 \text{ mm}^2 \gg l_N^2$ ) but it is difficult to see how this can be relevant for the bottom arm which is the *current* drain. The argument however applies to the spin leaking through the voltage probes.

An expression which takes into account spin flow in the bottom arm is actually easy to derive in line with the calculations done with the cross geometry.

Referring to Fig. 24 we consider a three arm geometry (arms  $I - III$ ) with two ferromagnetic terminals  $F1$  and  $F2$  with current flowing from  $F1$  to  $III$  through  $I$ . We allow a finite size for arms  $I$  and  $II$  ( $L_1 = l_1 l_N$  and  $L_2 = l_2 l_N$ ) for a more general expression. We find:

$$\Delta R_{nl} = 2R_{eff} P_{eff,1} P_{eff,2} \quad (\text{B2})$$

where:

$$R_{eff}^{-1} = \left[ \frac{1}{R'_N} + \frac{1}{R_{eff,1}} + \frac{1}{R_{eff,2}} \right] \quad (\text{B3})$$

The effective polarizations  $P_{eff,i}$  for terminals  $F1/F2$  and spin resistances  $R_{eff,i}$  for arms  $I$  and  $II$  are defined as in the bulk of the paper (with  $X_1 = X$  and  $X_2 = Y$ ):

$$R_{eff}^{-1} = R_N^{-1} \left[ \frac{R_N}{R'_N} + \frac{\delta_1^-}{\delta_1^+} + \frac{\delta_2^-}{\delta_2^+} \right] \quad (\text{B4})$$

We have allowed for different cross sections  $A_N$  and  $A'_N$  along  $x'Ox$  and  $Oz'$  so that the spin resistance along  $x'Ox$  is  $R_N = \rho_N^* l_N / A_N$  but along  $Oz'$  is

$$R'_N = \rho_N^* l_N / A'_N.$$

The total spin resistance is easy to interpret: it corresponds to parallel addition of the spin resistance of the three arms with spin resistance  $R'_N$  for arm  $III$  and spin resistances  $R_{eff,1}$  and  $R_{eff,2}$  for arms  $I$  and  $II$  respectively.

Suppose we neglect arm  $III$  spin resistance so that:

$$R_{eff}^{-1} = \frac{1}{R_{eff,1}} + \frac{1}{R_{eff,2}}. \quad (\text{B5})$$

One then recovers Fert-Lee and Hershfield-Zhao expression [Eq. (B1)] when  $l_1 = l_2 = l/2$ , which therefore does imply a neglect of spin leakage to the bottom terminal.

When is it allowed to do so? The condition is:

$$R'_N \gg (R_{eff,1}, R_{eff,2}) \quad (\text{B6})$$

The effective resistance varies between  $R_N$  and  $R_F + R_c$ ; the condition then reduces to:

$$R'_N \gg (R_{F1} + R_{c1}, R_{F2} + R_{c2}, R_N). \quad (\text{B7})$$

which is favored by small cross-section  $A'_N \ll A_N$ . Note that the condition  $R_N \ll R'_N$  is however not enough to recover Eq. (B1).

So more generally when the condition in Eq. (B7) is not met, one gets (when  $l_1 = l_2 = l/2$ ):

$$\begin{aligned} \Delta R_{nl} = & 8R_n \widetilde{PR} \widetilde{PR}' \\ & \times \{2(X+1)(Y+1)\exp l - 2(X-1)(Y-1)\exp -l \\ & + \frac{R_N}{R'_N} [(X+1)(Y+1)\exp l + (X-1)(Y-1)\exp -l + 2(XY-1)]\}^{-1} \end{aligned} \quad (\text{B8})$$

which is significantly different from Eq. (B1). Let's define the spin mismatch  $Z = R'_N/R_N$ .

At small distance where the signal will be largest:

$$\Delta R_{nl} \sim \frac{XY}{X^{-1} + Y^{-1} + Z^{-1}}.$$

The scale will be set by the smaller of the spin impedance mismatches. If anyone of them is in the transparent regime, one will recover the scaling characteristic of open systems:  $\Delta R_{nl} \leq R_N$  as it should be.

We have neglected in the above spin leakage in the voltage probes but it is easy to generalize the present considerations to include it, should it become experimentally relevant.

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